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The Role of the Mathematics Teacher in Our Defense Program

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MY TOPIC—"The Role of the Mathematics Teacher in Our Defense Program"—is so broad as to be almost all inclusive in its scope. With such a topic one might select almost any phase of mathematical education at random and prepare a pertinent paper. In every aspect of the defense program—financing, preparation, productions and use of defense mechanisms—one finds ample evidence of the importance of mathematical education. How then is the topic to be delimited?

There is no need to discuss here the need for loyalty of thought and action on the part of the mathematics teacher. These are to be expected of *him* as well as of every other citizen. The mathematics teachers of the country need to be told their duties as citizens neither more nor less than other citizens. Those of us who are called in the draft, who are needed for defense industries, or who, for some reason or other, enter some branch of the armed forces are doing only the same as many other citizens in this time of national crisis. As citizens we have a part to play and I believe we will play it to the best of our ability.

Even as the mathematics teacher has certain obligations as a citizen, so has he certain additional obligations as a teacher. We are all aware of the nature of these obligations and there is little need for further discussion of them. We all recognize our responsibilities with respect to the student, the school, the community, and the country at large. These responsibilities are not unique with the mathematics teacher but are the obligations of all who teach. During the last decade much paper was used in the discussion to the last detail of the nature of these responsibilities. There were many differences of opinion on the matter. If the public press is to be credited, many of these differences have been dissipated under the pressure of public opinion. In times of national crisis the responsibilities of the people receive more public attention than in calmer days. Our rights are safeguarded in the constitution. We must formulate our own Bill of Obligations.

While the two aspects of teacher responsibility mentioned above are important they are not our present concern. We must now examine our *unique* role in the defense program. What are the problems peculiar to our field? What are the things which we can do better than other teachers? How can the forces of democracy be strengthened by our efforts? What part of all that we teach or could teach would be of most

worth in this time of crisis? If we can answer these questions we will know the part we must play—the obligations which parallel our privileges.

There are three chief aspects of the defense program. We are now concerned with the development of an army, a navy, and an air force. These fighting forces we may well call the first line of defense. Modern warfare and its use of manufactured products make it increasingly apparent that our second line of defense is the plant which produces the materials of war. The third line of defense is the public.

The Mathematics teacher has certain responsibilities in training men for the first line of defense, for producing men with enough mathematical background to be able to operate efficiently in the armed services. The teaching involved here is not so different from that we have been doing in the past. Perhaps it would be safer to say it is not different from that which has been considered desirable in the past. We have also certain responsibilities in training key men for the second line of defense—chemists, physicists, engineers, and other men skilled in the sciences. Here too our problem remains much the same as it has been. These men are our college graduates. Much of our past concern has been with their specialized training. We are not likely to make immediate and rapid strides in its improvement.

Our new and great responsibility lies in two fields, in training men for the skilled and semi-skilled jobs in our second line of defense and in the training of the public. For ultimately the best defense of democracy *is* an enlightened public which knows and appreciates its opportunities and recognizes and accepts its responsibilities.

The future of mathematical education depends upon our acceptance of our obligations to these two groups. We are now riding the crest of a wave. The present crisis has caused an increased demand for factual knowledge and the ability to apply it. This, in turn, has brought about an increased demand for mathematical education. But an examination of this demand clearly indicates the necessity for a reconsideration of our philosophy with respect to the objectives of mathematics for the two groups mentioned. It is important to bear in mind the fact that the remainder of this discussion deals with these two groups only and does not refer to the pre-college student.

Within the past few weeks I have had the opportunity of discussing the mathematics program of the schools with several men who are closely associated with our defense effort. Of those to whom I talked Lieutenant du Chesne of the Middletown Air Depot was the most kindly disposed in his criticism of present practice in mathematics teaching. He told me that the trainees under his supervision had had plenty of mathematics but that most of it was valueless because students could not apply it. He implied that there had been too much mathematics spread too thinly in the high school education of these boys. Others have been of virtually the same opinion. This, then, is the opinion of men who are training skilled workmen for defense. They would like those who come to them to have more elementary mechanics, blue-print reading, approximate computation, and the like. Thus it appears that the chief demand for mathematics in this field today is based upon the fact that it constitutes a basic tool in many fields. More and more students are approaching mathematics with this attitude. What are the implications of this attitude? There can be no doubt that they are important.

We have tended, in recent years, to discourage the idea that mathematics is a "tool subject." Is there good reason for this? I think not. A competent workman has the highest regard for his tools, particularly those which he finds most useful. It seems paradoxical that we should tend to disclaim the uses of mathematics in favor of their imponderable values. Is it possible that in the past we have been too much concerned with imponderables in our field and too little interested in its "tool"

values? Must we continue in this attitude or can we link the two objectives successfully?

Let us examine the present standard curriculum. Secondary mathematics at the present time consists of a six year sequence more or less undifferentiated with respect to the needs and abilities of the students subjected to it. It is true that some schools make special provisions for the bright and dull student. Too often such provision consists of doing either more or less of the same thing as the average group. The differentiation tends to be quantitative rather than qualitative. In addition to this fact we must recognize that the student has only two choices at any point—to continue or drop the sequence. If he continues we know the path he will follow. If he drops the sequence he will take no further mathematics unless he enters the commercial or vocational department. There certain additional demands will be made of him. Such demands will probably be entirely independent of the mathematics department in which he has received his earlier training. Indeed, in some states, the law provides that the mathematics teacher may not teach commercial or vocational mathematics without a special license.

With full recognition of the high degree of differentiation needed in the vocational and commercial fields it nevertheless seems reasonable to conclude that there is a common body of mathematical knowledge which all students need. At the present time we are either presenting this work at at the wrong time or not at all.

Ther may be some present who have an abiding faith in the present set-up. But it appears certain that we can neither hold nor prepare students properly with the present curriculum. Must we therefore discard algebra, plane and solid geometry and trigonometry? Certainly not! But we must revise our curriculum with an entirely different emphasis, keeping those parts of the present curriculum which will be valuable and judging their value primarily by pragmatic standards.

Must we then give up our more or less abstract objectives for those which are concrete? To say that we must implies that those parts of mathematics which are practical in their value have no abstract value. This is not the case. The student will obtain more knowledge of the abstract aspects of mathematics—of its beauty, its clarity, its generality—through its use in connection with understandable situations than through its study as a discipline without applications. It has often seemed remarkable that we should expect students to receive some of the abstract benefits which we have claimed for our subject when they were weak in even the simple practical applications of the material.

Nor is this the only problem which confronts us. We are not only faced with the necessity of weeding out but also with the necessity for transplanting much of our material. Many of the topics of the junior high school are grossly misplaced on the assumption, once valid, that the student may drop out of school at the end of the ninth year. Whether or not such a reason is now sufficient for the inclusion of any topic at this level needs no discussion.

With the foregoing discussion in mind let us attempt to answer the question posed earlier—"What part of all that we teach or could teach would be of most worth in this time of crisis"?

The primary purposes of instruction in mathematics have not changed greatly since the 1923 report said "The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and action which will make these powers effective in the life of the individual."

This is a reasonable statement of the

purposes of the teaching of mathematics with which no thoughtful person could take issue. It remains as valid today as it was in 1923. It is as valid in today's crisis as it was in those more peaceful times. And the one phrase of the whole statement which makes it valid is "which will make these powers effective." Without the inclusion of this phrase the statement is too all-inclusive to withstand critical examination. And yet, have we placed enough weight on this particular phrase in our construction of the mathematics curriculum? Has effectiveness been our fundamental criterion for the inclusion of subject matter in our courses of study? We must admit that it has not, for if it has been how can we account for the fact that we are graduating so many people who are numerically illiterate? How can we account for the criticisms on the part of employers which are largely concerned with the numerical inefficiency of young job holders and for the criticisms such as those mentioned above which are advanced by those concerned directly in the defense program?

We cannot hope that mathematics will become effective in any real sense in situations which are not familiar to the student. It is equally true that we cannot compile an exhaustive list of situations with which the students are familiar. But somewhere there must be fertile soil where the seeds of mathematics can be planted. It is found when the need to solve a problem is combined with a knowledge of the skills necessary to a solution and the ability to apply these skills. Then and only then does mathematics become effective. If the situation arises and the skills are not available or if the skills are available and the situation does not arise we are on barren ground. Too much of this barren ground is responsible for our failure with the two groups under discussion.

Unquestionably, the most important applications of mathematics in the life of the average individual are in the field of arithmetic. Yet we slight instruction in arithmetic after the eighth year and in the seventh and eighth years we teach much of the work which will be of greatest value to the student after he graduates, teach it at a time when it is far beyond the realm of his experience, incapable of immediate application, and doomed to deterioration over a period of at least four years. I am referring to our junior high school work in interest and investment, taxation and insurance, budgeting and banking. Even the best of teaching in these topics cannot produce at this level an understanding of these topics which will survive four years of stagnation. Most of the graduates of our high schools leave our hands with only the sketchiest idea of the financial aspects of their future life because we are now teaching them so that those who leave school at the end of the ninth year may leave with these understandings which are so necessary to their future. And how many of them are leaving at the end of the ninth year? What price our criterion of effectiveness in this connection?

The solution is obvious. Remove these subjects from the seventh and eighth years and place them in required courses in the twelfth year. The increased maturity of the students combined with the immediacy of their need for the information to be acquired in such a course will at least allow for a greater dividend on the instruction than is now obtained. A course of this kind is our greatest need at the present time. It will go farther toward the production of an enlightened citizenry than any other we could offer. A reasonable understanding of the financial organization of the country, of the financial responsibilities of the individual, and of the economic and financial status of the citizen of the United States as compared with the citizens of other countries should be a good defensive weapon in the hands of our democracy.

A course such as this in the twelfth year would have the additional advantage that it would give the student further training in arithmetic. And at this level the train-

ing would have as its objective reasonable speed and absolute accuracy. This would be possible here since it would be merely setting the standard which would be demanded by prospective employers the following year. Many employers are outspoken in their demands for such standards on the part of those they employ and are critical of the schools for failing to meet such standards. And yet, is it remarkable that we fail to achieve these standards when we absolve students from responsibility for mathematics at the end of the ninth year and in our training up to that time demand only "reasonable speed and accuracy"? By giving this simple training for positions having limited but specific requirements, we will be accomplishing still more in the program for national defense.

So much for arithmetic. What of algebra? Does the algebra we teach in the junior high school function in the life of our students in the way it should? Is it "effective in the *life* of the individual"? One can find plenty of reason for doubting that it is. Wherein lies our failure in this respect? Mainly in the fact that we teach those things which can be applied rather than those which need application. We search for applications of the mathematics we teach rather than searching the ordinary situations for their use of mathetics and then teaching the mathematics. Even in our elementary algebra of the formula, the equation, the graph, and the directed number, we teach much that is of little value. The formula, the equation and the graph are of particular importance and need much attention. But in directed numbers do we do much for the ordinary student that will be of real value later? Do we not cloud the issue with rules and regulations, with talk of thermometers going up and down, increasing debts, and decreasing liabilities, and other such devices in an attempt to teach the operations with signed numbers which to most of our students are not understandable? Do students at this level ever really learn to differentiate between the minus sign as an operator and as a sign of direction? Perhaps some of them do but not a great many. But there is a large field for instruction in elementary algebra (including some statistics) without approach to the signed number. For the general group I think that such a course could be developed with profit and that the time saved in the work in signed numbers could be profitably devoted to laboratory work and the experimental development of much of the work which is now taken for granted. The work, to be of significance, must be developed from real rather than vicarious experiences. Only to this extent will it be valuable. The non-preparatory student must have a reason other than the demands of college preparation for those things which are asked of him.

In geometry it is unquestionably true that the student should derive some knowledge of the nature of a proof, of the nature of proofs in life situations, and of some of the more important geometrie facts. It seems that he will understand the meaning of proof more thoroughly from the development of a single unit which he can look upon as such, rather than from the development of a complex discipline. If the unit is, for example, that leading to the Pythagorean theorem, he can see it as a whole, shorn of all irrelevant and noncontributing facts. The nature of the logical structure is laid bare for his examination. He can derive from such a unit all those concomitant values which we advance as important contributions of the teaching of geometry. What, of all the rest of geometry increases the number or value of these concomitants? Is it reasonable to state that work in geometry for this group beyond this point which has other than factual objectives is invalid? Are we, by perpetuating the whole logical structure of Euclidean geometry, developing further those "powers of understanding and analysing relationships of quantity and space" referred to in our objectives?

Much the same criticism may be ad-

vanced for trigonometry. Do our needs extend beyond what we now teach as numerical trigonometry for the average student? Most of us will agree that they do not. We require only numerical trigonometry in our recommendations for the average student.

Thus it might appear that from my point of view the responsibilities of the mathematics program as outlined here consist of little more than the present junior high school requirement.

As to subject matter this is probably true. There is no reason to believe that the present emergency demands more mathematical training than has been deemed desirable in the past. But our real need is not for more subject matter. It is for more applications in fields related to defense training. There is much material now included in the curriculum which, in the interests of the student and of the national defense, could be replaced by materials which can be used effectively.

What then, is the role of the mathematics teacher? It is one of critical examination of both the materials of teaching and of methods of teaching. It is a task of constant searching—not for more applications of mathematics, not for reasons for the inclusion of more work in the curriculum, not for reasons to perpetuate the present curriculum—but, for places where mathematics can serve the cause of democracy, for places where it must be applied, for effectiveness in the life of the pupil.

It was indicated earlier that the place for this effectiveness is in the ordinary life of the average citizen and in the occupation of the skilled worker. Let us vision for the average student more mechanics, more blue print reading, more measurement, more planning and design, more understanding of the ordinary financial aspects of average existence and we, as teachers, will be making a great contribution to the defense of our democratic form of life.

And if there are some who are of the opinion that the role of the teacher as outlined in the development of this discussion is not specifically for the defense of democracy, that it is much the same as might be desirable in less troublous times, allow me to point out that the defense of democracy must be a continuous effort. Democracy must stand through what it offers in ordinary times, for in a democracy strength is in the hearts of the people and not only in the strength of the fighting forces. And if, further, there are those who feel that the outline submitted fails the cause of mathematics in any way in failing to emphasize sufficiently the imponderables in favor of the tool values of mathematics allow me to quote the following lines:

"The world has constant use for boys like this
Whose work's not fast and loose nor hit or miss,
Who seek with heart athrob where they may dwell,
These boys who know their job and do it well."

and to point out that in years to come our contribution to the defense effort will be judged by the number of boys of this kind we graduate from our schools and by no other standard.

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THE Museum of Science and Industry, Jackson Park, Chicago, Illinois has changed the list price of *The Tree of Knowledge* (in color) from 20¢ to 25¢ in the Museum and from 25¢ to 30¢ prepaid by mail. Small copies (in black and white) for class use can still be obtained for 5¢ each postpaid (50¢ per dozen) from THE MATHEMATICS TEACHER, 525 W. 120th St., New York, N. Y. Do not send us money for the colored poster—Editor.

Progress Report of the Subcommittee on Education for Service of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America

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1. *Introduction.* A report of activities and recommendations was recently presented to Professor Marston Morse, Chairman of the War Preparedness Committee, by its Subcommittee on Education for Service. The following extracts differ from the complete report through the omission of certain recommendations which at the moment are not sufficiently matured to be appropriate for publication. All curricular recommendations are included in the following paragraphs.

The active members of the Subcommittee, who subscribe unanimously to the report are as follows: R. S. Burington; H. B. Curry; E. C. Goldsworthy; F. L. Griffin; W. L. Hart; M. H. Ingraham; E. J. Moulton.

2. *Activities.* Members of the Subcommittee and a few other cooperating mathematicians reviewed or inspected an extensive list of books and pamphlets of a mathematical nature which are employed as text material in service schools of the Army and Navy and in the Civil Aeronautics Program. Personal contacts were established by the Subcommittee with various regular officers of the Army and Navy, teachers in the Civil Aeronautics Program, representatives of the field of secondary mathematics, and certain individuals of experience and reputation in the field of vocational education. In spite of these efforts to obtain an objective foundation for opinions and recommendations, it must be definitely admitted that the need for speedy action, as well as the intangible nature of certain features of the situation, did not permit the Subcommittee to adopt an entirely objective approach. In evaluating evidence and formulating opinions, the Subcommittee was aided by the fact that certain of its members had had military experience during the First World War.

3. *Statement of general viewpoints.* Mathematicians who are interested in the contacts between their field and emergency problems of national defense and industry should guard themselves against attaching too much importance to the most advanced mathematical aspects of the situation and also too little significance to the elementary or intermediate mathematics which is of use in many directions. It should be realized that, in our nation, which in the past has always been geared to a peacetime economy, with only brief intermissions when military affairs were reckoned of importance, there is likely to be a large element of surprise in the public reaction to information that military science, in most of its important branches, is mathematical in nature. However, in such a statement we do not imply that the mathematics involved is necessarily of *advanced* type. Also, we must be alert to recognize the presence of methods or theory in military and naval science which involve a mathematical *background* for intelligent appreciation, even though, superficially, no mathematical techniques are employed. A similar statement can be made concerning an evaluation of the mathematical needs of industrial workers, below the level of engineers.

In arriving at an estimate of the mathematical background which is *desirable* for

workers in government and industry, and for officers and enlisted men in the Army and Navy, we recognize the validity of the following pedagogical viewpoint. *In order that an individual may be able to use effectively any particular body of technique, his school training should extend a reasonable distance beyond the level of difficulty at which he will apply the technique.* Thus, if we wish to prepare a student so that, later, perhaps after some review, he can use *elementary algebra*, he should be exposed to *advanced algebra*, or to some other mathematical subject with elementary algebra as a prerequisite. This pedagogical viewpoint is at variance with emergency actions which would attempt to give men the bare minima of mathematical techniques necessary for a *formal* approach to their applications. An emergency justifies *any* remedial action, but our efforts should be directed toward making it unnecessary to use hazy emergency shortcuts to mathematical procedures. With our widespread democratic system of secondary and collegiate education, our nation is justified in demanding that we should always have on hand a relative *surplus* of people with mathematical training through substantial secondary mathematics and also a surplus with elementary college training in the subject.

At this point it is pertinent to remark that, in the remainder of this report, any apparent omission relating to content or training at the graduate level is due to our decision that the omitted matter falls more properly in the spheres of other subcommittees of the War Preparedness Committee.

4. *Recommendations concerning mathematics for those engaged in non-military activities.* The importance of military aspects of the present national emergency should not cause us to lose sight of the equally important mathematical features of the normal and emergency activites of government, the various learned professions, and industry. We observe an enormous expansion in the aircraft and other munitions industries, a continuous drain on the national supply of skilled workers due to Army and Navy calls for enlisted specialists, and the extensive statistical and accounting work associated with our national economy. It is our opinion that these features of the present situation, as well as general underlying trends independent of the emergency, create a need for an increased supply of young men and women with training in mathematics through various levels beyond the junior high school grades. We believe that skilled workers in mechanical industry should have, in their backgrounds, substantial secondary mathematics through the stage of computational trigonometry, and at least an intuitional and *sketching* acquaintance with the fundamental notions of solid geometry. Also, we recommend that increased numbers of men and women should be trained at least through substantial secondary mathematics, to create a reservoir of suitable candidates for positions demanding mathematical skill and for the professions where advanced mathematical knowledge is of advantage. In particular, it would be desirable to have numerous women trained through the stage of elementary mathematical statistics, for the use of government, the professions, and industry. We believe that these recommendations for mathematical training, in so far as they relate to secondary or elementary college mathematics would not be harmful in connection with other educational objectives dissociated from the field of mathematics.

5. *Evaluation of the mathematical needs of the Army and Navy.* If we ask what the Army and Navy would desire as mathematical training for officers under *ideal* conditions, we obtain a sufficient answer by observing the intensely mathematical and technical nature of the curricula in the academies at West Point and Annapolis. The desirable level of training could be maintained during the present expansion of the Army and Navy only if all the officers, particularly in certain branches, were

required to be engineers. We may assume that this ideal obviously is impossible of attainment, if we admit the truth of many authoritative statements that a shortage of engineers exists at present even for the needs of industry alone, apart from the requirements of the Army and Navy. Our evaluation of the mathematical needs of the Army and Navy will be made in the light of the emergency situation, with a subjective view as to the levels of training which are possible of attainment if full advantage is taken of our extensive educational system. Moreover, we shall give no consideration to mathematical features of the curricula at West Point and Annapolis, because the graduates of those schools obviously have fine training for their activities. We are interested in the mathematical backgrounds of all others, officers or enlisted men, outside of the commissioned personnel of the Regular Army and Navy, who are in or who will enter our Army and Navy. We shall omit mention of various branches of the Army whose activities obviously are non-mathematical. In summary, we believe that the following specifications of mathematical training for *officers* give *minimum* levels, if our Army and Navy are to be intelligently led. The training specified for various enlisted men may exceed the true minimum levels but probably are the *desirable* levels, and we believe that they can be attained. However, these training goals for both officers and enlisted men will *not* be attained unless special efforts are made by the high schools, colleges, civilian centers of adult education, existing service schools of the Army and Navy, and directors of education in the Army and Navy outside their existing schools.

5.1. *Infantry.* Even this supposedly nontechnical branch of the Army places demands on mathematics. All enlisted men find use for arithmetic and intuitional geometry and would be benefited by the content presented in modern courses in mathematics for grades eight and nine. The officers, non-commissioned officers, and privates first-class should have familiarity with elementary geometry to permit map reading and construction, appreciation of contour designations, and the use of coordinate systems. They should be able to study intelligently the mechanical drawings associated with the rifles, light antiaircraft guns, motorized equipment, and other materiel assigned to them. In brief, these officers and the upper groups among the enlisted men should have as substantial a background in mathematics as we consider desirable for skilled workers in mechanical industry. In addition, the officers would find it useful to have an acquaintance with the notions of probability and probable error as met in elementary statistics so as to appreciate the theory of gunfire as applied to fire by the infantry and either opposing or supporting fire by artillery.

5.2. *Coast Artillery Corps.* This exceedingly mathematical branch controls all artillery for seacoast defense, high altitude antiaircraft artillery, and mobile artillery of heavy caliber. The officers of this corps have to perform the duties of surveyors on some occasions, and they deal with very complex optical instruments, motorized equipment, and complicated guns. These men should have very strong training in mathematics; in fact, we hesitate to specify training short of that possessed by graduates of an engineering college. But, as a minimum, these officers should have passed through advanced high school algebra, computational plane trigonometry enough spherical trigonometry for its typical applications in surveying, and the elements of solid geometry. Also, they should have an acquaintance with the notions of probability and probable error for appreciation of the theory of gunfire. In addition, a substantial number of the enlisted men should be as well qualified mathematically as the officers, so as to provide intelligent personnel for technical groups and to permit the training of enlisted understudies for all the officers as insurance against the effects of casualties.

It would be desirable if practically all the enlisted men had the mathematical background which we consider suitable for skilled workers in mechanical industry.

5.3. *Field Artillery.* We make the same minimum stipulations for mathematical training in this branch as in the Coast Artillery Corps, with omission of mention of spherical trigonometry for the officers, and with less insistence on the need for mathematical backgrounds in the case of the enlisted men.

5.4. *Signal Corps.* The officers should be electrical engineers and the enlisted men should have the mathematical training suitable for skilled workers in mechanical industry.

5.5. *Ordnance Department.* It needs various specialists, among both the officers and the enlisted men, with highly mathematical backgrounds such as possessed by engineering graduates or college majors in mathematics and physics.

5.6. *Ground Force of the Air Corps, in the Army and Navy.* The ground service requires many engineers, men with extensive college mathematics and physics especially for the meteorological section, and a large number of enlisted men and officers with the mathematical background suitable for skilled mechanical industry.

5.7. *Pilots or navigator-gunners in the Air Corps of the Army and Navy.* They should be acquainted with plane trigonometry, the elements of solid geometry, and an introduction to spherical trigonometry. They need this content for the study of navigation, elementary aerodynamics, bombing, meteorology, and various other technical subjects. The importance of space concepts and physical reasoning in three dimensions in such subjects adds to the importance of solid geometry for this group of men. Very substantial manipulative algebra is needed by them and we presume that this is included as a prerequisite for the trigonometry course. Moreover, they should have acquaintance with the notions of probability and probable error to aid in the appreciation of certain aspects of the theory of bomb dropping.

5.8. *Officers of the Navy, outside of its Air Corps, and officers of the Merchant Marine.* We hesitate, again, to specify training short of a college degree in some field of engineering. As an emergency minimum, however, we recommend that these men should have the mathematical background specified for pilots in the Air Corps, with additional emphasis on algebra and spherical trigonometry. We include reference to officers of the Merchant Marine in our remarks because of its general importance and also its complementary relationship to the Navy in time of war.

5.9. *Enlisted men in the Navy.* They should have the mathematical background desirable for skilled workers in mechanical industry. In addition, a substantial number of these enlisted men should be as well qualified mathematically as the officers, to provide intelligent personnel to serve in technical groups and as understudies for officers.

6. *Conclusions drawn from results of the program of reviews of books of a mathematical nature used by the Army, Navy, and Civil Aeronautics Authority.*

6.1. By and large, the mathematical exposition in these books is satisfactory, particularly when we take account of the fact that they are intended for readers with a minimum technical background. The reviews do not justify us in calling for prompt revisions of any of the existing text material, although various criticisms of present expositions could be made.

6.2. We believe that any one of these books can be well appreciated by a reader who has a proper mathematical background, in accordance with our preceding recommendations, and a suitable teacher. This opinion permits the Subcommittee to place its main emphasis *first*, on the preceding recommendations for mathematical backgrounds, and, *second*, on an analysis of the effects of these recommendations on

plans for instruction in the civilian and military educational system.

7. *Recommendations concerning the field of secondary mathematics.*

7.1. In the secondary field, it would be very undiplomatic and harmful if the national emergency were taken as an excuse for a violent attack on certain curricular trends, even though weaknesses of some of these trends may become apparent when they are analyzed under the searchlight of present national necessities. We consider it best to state mathematical objectives without stipulating the pedagogical details to be involved in their attainment.

7.2. The National Council of Teachers of Mathematics and all organized bodies of teachers of mathematics at the secondary level should advertise the utility of mathematics in industry, government, the professions, and military science.

7.3. In high schools it should be advertised that the Navy R.O.T.C. and the Coast Artillery groups of the Army R.O.T.C. in colleges require trigonometry as a prerequisite and that they should require the elements of solid geometry and spherical trigonometry. Also, it should be emphasized that these subjects can be studied efficiently in high school.

7.4. In the junior and senior high schools, each boy and girl *of sufficient mathematical aptitude* should be urged by his advisers to observe that the study of mathematics through the stage of trigonometry and some solid geometry may serve as a distinctly patriotic action.

7.5. We recommend that, in connection with emphasis on so-called *socialized* aspects of secondary curricula, a liberalized definition of *socialized mathematics* should be adopted for students at all ability levels, in contrast to more narrow definitions which give unique prominence to business applications and consumer interests. In the liberalized definition we would emphasize that content with military, scientific, professional, and industrial uses is of a most socialized nature. Also, from the standpoint of a high school student of intelligence, *classical* mathematical content may be very "*socialized*," in a true sense, even though the content possesses only *delayed* utility, as contrasted with *immediate* utility in the student's experience.

7.6. The military, industrial, and scientific utility of a considerable quantity of space intuitions and at least a little spherical trigonometry, causes us to recommend that the high school work in solid geometry, both intuitional and demonstrative, be given more prominence than in recent years. The classical course in solid geometry might be modified by replacing some of its content with a treatment of the elements of spherical trigonometry. Or, the intuitional and demonstrative plane geometry presented in grades eight through ten might be modified to include sufficient material from solid geometry.

7.7. We *strongly* recommend that a *single* set of courses be used in any high school for students of *appropriate ability* in attaining desired ends relating to industry, military service, or future collegiate education. We recommend this single treatment rather than separate curricula, some designed to fit men for industry or military service and some planned for those who will delve more deeply into mathematics and related fields in college. In the case of superior students, substantial mathematics, fitted to their intelligence, is likely to serve them better, whenever they will use mathematical content, than specifically pointed vocational mathematics or military mathematics. Thus, we argue against a curricular division in mathematics among secondary students which would be based on their present economic status, their *momentary* expectations about attending or not attending college, or *transient* vocational preferences, and we advise instead a curricular division based on the *intelligence* of the students.

7.8. As a temporary measure, we recommend that boys of intelligence who now are in grades eleven and twelve and who have previously omitted substantial mathematics, should be offered an abbreviated

treatment of logarithms, plane trigonometry, intuitional solid geometry, and perhaps an introduction to spherical trigonometry, to train them for their practically certain entrance into skilled industry, the Army, or the Navy.

7.9. We advise the evening schools in cities to give new emphasis to courses in advanced high school mathematics through the stage of trigonometry.

7.'10. We advance the opinion that a shortage of engineers and physicists is at hand. This should be brought to the attention of boys of mathematical ability in the high schools; if these boys extend their exposure to secondary mathematics but later fail to become engineers or physicists, their mathematical training will have sufficient general utility to justify our recommendation.

7.'11. At all stages of secondary mathematics we recommend emphasis on applications. However, the teacher and student should not anticipate that all these applications, or even a majority of them, will be of intrinsically natural types. The pedagogical aim in this connection should be to convince the student that mathematics has not only important cultural and theoretical sides but also is intensely *useful* in our civilization; the applications, artificial or real, should give the student experience and confidence in applying general mathematical techniques as auxiliaries in related fields.

8. *Curricular recommendations at the college level.*

8.1. We wish to re-emphasize an early recommendation of the War Preparedness Committee by suggesting that as many college teachers of mathematics as possible should carry out measures of self-instruction in one or more of the following fields: Hydrodynamics; Aerodynamics; Meteorology; Probability and Statistics; Computation; Industrial Applications of Mathematics; Exterior Ballistics; Navigation; Artillery Fire Control and Orientation; Cryptanalysis. An enlarged background in these fields would be useful to the teacher of college mathematics at any time and, at this moment, would aid him in introducing problems of emergency interest into his routine courses. Also, such self-instruction would prepare the teachers for emergency use in directions where their talents would be of advantage. Those interested in such self-instruction should consult the bibliography published in the Report of the War Preparedness Committee in the *Bulletin of the American Mathematical Society*, page 713, 9:46:1940.

8.2. We consider it justifiable for the Department of Mathematics in any college to introduce undergraduate courses in hydrodynamics and aerodynamics, and courses in meteorology in conjunction with the Physics Department, in case such courses are not being given by other departments.

8.3. We recommend a course in Exterior Ballistics, with a prerequisite of at least elementary calculus and a first course in differential equations, for consideration in the organization of work in applied mathematics.

8.4. We suggest that, in the senior college years, a combination field of major concentration in mathematics and its applications be provided by the Department of Mathematics and the Departments of the Physical Sciences, perhaps in collaboration with the Departments of Engineering if they are present in the college. Such an undergraduate major might involve some sacrifice of pure mathematical content as compared with a narrow major in mathematics. If the curriculum were organized on a five-year basis, through cooperation between Mathematics, Physics, Chemistry, and Engineering, the curriculum would offer training which could be very useful in industry.

8.5. We wish to caution Departments of Mathematics to avoid indiscriminant introduction of elementary courses in *war* mathematics. In particular, we do not consider it desirable to suggest any special course pointed at Army or Navy service which does not have at least plane trigo-

nometry and substantial manipulative algebra as a prerequisite. This opinion is due to our conviction that the classical material just mentioned is more valuable than a preliminary exposure, with a weak foundation, to military and naval applications in advance of their later study while in service in the Army or Navy. Moreover, we observe with satisfaction that the Army and Navy, in certain directions, are giving explicit credit in priority ratings to men with credit in *classical* courses in trigonometry and advanced algebra, and also in more advanced mathematics. Hence, for any given group of students, we recommend that a Department of Mathematics should carefully investigate the relative advantages of *more classical mathematics* as compared to any *emergency* course in war mathematics before it is introduced.

8.6. For certain special groups of students, we recommend consideration of the following emergency courses, with plane trigonometry and college algebra as a prerequisite:

8.61. For students who are fairly sure that they will enter the Air Corps, the Navy, or the Merchant Marine: a course in Navigation, including necessary spherical trigonometry and solid geometry.

8.62. For students who are fairly sure to enter the Air Corps, the Navy, or the Coast Artillery Corps of the Army: a brief course, to be offered two hours per week for three months, in the elements of Solid Geometry and Spherical Trigonometry.

8.63. For students who plan to enter the Army or the Navy in any capacity and who wish a review of probably useful material with moderate additions: a semester course, to be offered three or four hours per week which will present, *first*, a review of geometry, necessary algebra, logarithms and trigonometry, with emphasis on its numerical aspects, and, *second*, new material relating to solid geometry, spherical trigonometry, and probability as involved in the theory of gunfire and bombing. We recommend this semi-review course for use in college extension curricula and in evening schools, as well as in daytime college work.

8.7. We suggest only the following moderate modifications in standard courses.

8.71. Considerably increased emphasis on applications and computational techniques; in this connection the teacher could bring to bear his extended background as advised in Section 8.1.

8.72. In Trigonometry, the inclusion of a treatment of mil measurement (the Army system of angular measure), emphasis on vector language and applications, and, for certain students, an expansion of the course to include some spherical trigonometry in colleges where that subject is not usually taught.

8.73. In College Algebra, expansion of the work in probability to include introduction of the normal probability curve, the general notion of a frequency curve and its probability significance, and use of the language of probable error.

8.8. We recommend that women who are studying college mathematics be given a course in mathematical statistics with at least freshman mathematics as a prerequisite, regardless of the individual educational objectives of these students. In addition to broadening their backgrounds for their peace-time vocations, this work in statistics might prove useful to the women in search for employment in present emergency activities.

8.9. We suggest that each Department of Mathematics canvass the situation of young men and women in its college who have mathematical talent, even though they may not be taking courses in college mathematics. The men should be made acquainted with existing opportunities for preference in the Army and Navy as the result of training in mathematics. The women should be informed of those semi-mathematical fields, for instance, business statistics, accounting, and drafting, where a continued drain on available man power may create openings for women.

8.10. Regardless of justified attitudes which cause colleges to avoid teaching ele-

mentary parts of secondary mathematics, we recommend that each Department of Mathematics should do everything in its power to aid interested college men, or high school graduates now in college, in learning secondary mathematics which will be of use to them in the Army or Navy. Such instruction could be offered through extension or correspondence courses, as well as in daytime classes, and should be arranged so as to avoid duplication with efforts of neighboring high schools.

8.'11. If a Department of Mathematics has taken all appropriate actions in accordance with the preceding Sections 8.2 through 8.'10, with particular attention paid to the cautions of Section 8.5, and in addition desires to provide a general elementary course in war mathematics, the following outline by Professor Griffin may be useful. This outline gives a rounded view of typical applications of mathematics in military and naval science which should interest men and women alike and which would have specific utility for men who will enter the Army or Navy. The content could be taught well only by one who has an extensive background gained either through actual service in the Army or Navy, or by study as suggested in Section 8.1.

WAR MATHEMATICS

(Three hours per week for one year; suggested by F. L. Griffin)

PURE MATHEMATICAL TOPICS

Chapter I. Preliminary Ideas.
Chapter II. Trigonometric Functions.
Chapter III. Logarithmic Calculations.
Chapter IV. Coordinates and Notions from Analytic Geometry.

ARTILLERY AND MACHINE GUN PROBLEMS

Chapter V. Position Calculations. Methods of locating a fixed target: by direct observation, indirect observation involving trigonometry; map location; sound methods.
Chapter VI. Ballistic Calculations. Initial firing data; adjustment of fire; probable errors; bracketing; effect of fire; velocity and angle of impact; penetration.
Chapter VII. Safety Zones and Dead Areas.
Chapter VIII. Barrage Fire.
Chapter IX. Theoretical Ballistics, Discussion of the construction of firing tables.

ARMY ENGINEERING PROBLEMS

Chapter X. Graphical Methods: Rates: Maxima: Work: Momentum.
Chapter XI. Statics: Bridge Structures: Cranes: Inclined Planes.
Chapter XII. Flexure of Beams: Suspension Cables.

AVIATION PROBLEMS

Chapter XIII. Principles of Flight. Stability; Equilibrium.
Chapter XIV. Bombing.
Chapter XV. Spherical Trigonometry and Navigation.

NOTE: For further information concerning the preceding course, consult Professor F. L. Griffin, Reed College, Portland, Oregon.

Distribution of the Report on Mathematical Education for Defense

The above report of the Subcommittee on Education for Service of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America has also been published in the AMERICAN MATHEMATICAL MONTHLY (July, 1941) and will appear soon in SCHOOL, SCIENCE, AND MATHEMATICS. A reprint of the report is being sent to the superintendent of schools in each city of over 2500 population. The expense of this distribution is being borne jointly by the American Mathematical Society, the Mathematical Association of America, and the National Council of Teachers of Mathematics. The publicity already provided for the report has placed it in the hands of the specified superintendents and all teachers of mathematics who belong to national organizations. In addition, it would be desirable for the report to reach the extremely large body of teachers of high school mathematics who do not belong to the National Council or to the Central Association of Science and Mathematics Teachers. Hence, reprints of the Report are being offered for sale while the supply lasts, *only in units of 25 copies each*, at $1.25 per 25 copies, delivered to the purchaser. Orders for these copies should be sent, with attached money order or check, payable to Professor W. D. Cairns, Secretary of the Mathematical Association of America, 97 Elm Street, Oberlin, Ohio.

Mathematics of Aviation

By Edwin A. Beito
University of Wichita, Wichita, Kansas

The rapid development of aviation in this country and the great interest shown by our young people should be a challenge to all teachers to make the most of its educational possibilities. The United States government has established pilot training schools in every section of the country, which are open to both college and non-college students. Thousands of private pilots are now being trained, and the future program calls for the training of 30,000 pilots per year. The Air Youth of America, an organization sponsored by the National Aeronautics Association and Winthrop Rockefeller, is forming Model Airplane classes in every city in the country, and the membership is increasing very rapidly. In my own city of Wichita, Kansas, more than five hundred boys and girls have joined the classes. These classes are held in connection with the public schools, after school hours.

Every phase of aviation from the designing of the plane to its flight from place to place is dependent upon mathematics. We, as teachers, should make use of the opportunity to popularize our subject so that it may become of greater service. It is our job to anticipate the mathematics needed by the coming generation, and change our courses accordingly.

Experience has shown that teachers in service are usually slow to respond to any changes toward practical mathematics because they wish to teach the type that they themselves have studied, and enjoyed. It is well known that teachers in general are helpless in applying their mathematical knowledge to practical situations. Teachers have told me that they are constantly stumped by pupils' questions on aviation. The time seems to be ripe for every teacher training institution to give some training in the fundamentals of aeronautics as a part of its course in Professionalized Subject Matter. Such courses are already being offered by some schools. The University of Minnesota gave an elementary course in general aeronautics for science and mathematics teachers last summer.

Mathematics of aviation includes aerodynamics, theory of flight, meteorology and navigation. Since we are today concerned cheifly with other-than-college preparatory students, I shall limit my remarks to the mathematics of navigation which promises to be of service to both college and non-college students who will operate, or hope to operate, airplanes in the future.

It shall be my purpose to discuss briefly the mathematical concepts of air navigation with a few remarks interspersed here and there.

At the outset, a mathematical understanding of latitude and longitude is necessary, and not the vague idea about imaginary lines showing positions on the earth. Latitude is the angular distance north or south of the equator, the angle being formed at the center of the earth. It can be demonstrated geometrically that the angle of the elevation of the celestial pole, which is near the North Star, is equal to the angle of latitude. It can also be shown geometrically that the angle between the zenith and the sun at noon on March 21 and September 22 is equal to the angle of latitude, and at other times corrections must be applied which are given in the American *Nautical Almanac*, the *World Almanac*, and other sources.

Longitude is the angular distance east or west of the Prime Meridian, the angle being formed at the axis of the earth. The longitude of a place is measured by the time required for the mean sun to travel from the 0° meridian at Greenwich, England, to the point in question. The time is measured by a chronometer, a time-piece which gives

Greenwich Civil Time. All ships and observatories are equipped with a clock which gives the same time as at Greenwich. If the mean sun is directly south, (which is at its greatest elevation), when the clock on the ship reads 4:00 P.M., or sixteen o'clock on the chronometer, the longitude is four times fifteen degrees or sixty degrees west longitude.

It is necessary to understand four different kinds of time in navigation, apparent solar time, mean solar time, standard time, and sidereal time. Apparent solar time is the time which was used by the Ancients, and is read from ordinary sundials. The apparent solar day is not the same length throughout the year, and since our clocks are run at uniform speed, it was necessary to invent mean solar time which is based upon the average speed of the sun throughout the year. When the average sun is south the mean solar time is twelve noon. The difference between mean solar and apparent solar time is called the Equation of Time, and is plus or minus depending upon whether the real sun is behind or ahead of the average sun. Standard time, and the mean solar time, are the same on the four principal meridians, 75°, 90°, 105°, and 120°, and a belt on each side of them theoretically seven and one-half degrees wide uses the same time as at the principal meridian. The belts are not regular, due to practical considerations, which produces some apparent discrepancies in time. For instance, the sun is due south as late as 12:56 P.M. during the year at Dodge City, Kansas, and as early as 11:10 A.M. at St. Andrews, Maine—a difference of nearly two hours. Knowing this, the reputed ability of the Missouri mule to tell the time of noon within a minute becomes more of a mystery than ever.

Sidereal time, which is measured by the earth's rotation, with respect to the stars, is often used in more modern forms of navigation, such as the Weem's System.

It is necessary for an airplane pilot to read maps accurately. To do this he must understand different types of map projections. A map projection is the systematic method of representing points on a sphere, upon a flat surface. The most common projection is the Mercator Projection. On it all parallels are of the same length and are spaced north and south proportionally to the secant of the latitude. The meridians are parallel lines perpendicular to the Equator and equally spaced. This projection is very valuable in navigation because the bearings are represented correctly; however, areas are greatly distorted. At 60° latitude the area is four times as great as an equal area at the Equator; and at 80° the areas would be increased by thirty-six times its actual size. For this reason people have formed erroneous ideas about their relative sizes of earth divisions. Knowing the mathematical basis of the projection the person will be able to adjust his impressions to see the true relationships. In geometry we pride ourselves upon studying size, shape, and position, but on our maps, we do not have the right size, or shape, and the relative positions are misleading. Neither is a straight line the shortest distance between two points! When I was studying geography in the eighth grade I thought that I had discovered a misprint when I saw a curved line across the Pacific representing a ship-lane, marked-shorter than a straight line between the two points. My teacher agreed that there was a misprint. However, the great circle is a curved line on a Mercator Projection, and represents the shortest distance between two points on the map.

An Equal Area Projection, represents areas correctly by spacing the parallels proportionally to the sine of the latitude, but the shapes are greatly distorted in the higher latitudes.

The Gnomonic Projection is a perspecrive projection upon a plane tangent to the sphere, with the point from which the projecting lines are drawn situated at the center of the sphere. A great circle is represented by a straight line on the chart, and every straight line is a great circle. For navigational purposes, the great circle is

transferred to the Mercator chart by determining the latitudes and longitudes of various points along the great circle line. When these points are plotted on a Mercator chart, the bearings of each leg may be obtained so as to very nearly follow the great circle.

A great circle on the earth contains 360° or 21,600 minutes. One minute of arc is equal to one nautical mile, and therefore the earth's circumference is 21,600 nautical miles. The circumference is about 25,000 statute miles, making a nautical mile equal to 1.15 statute miles. One degree of a great circle is 60 nautical miles, or about 69 statute miles. The distance from place to place can be found if the latitudes and longitudes are known. If one station is directly north or south of the other the approximate distance may be obtained by multiplying the difference in degrees of latitude by 69. If the direction is directly east or west, the distance will be 69 times the difference in longitude, multiplied by the cosine of the latitude. For example, at the 60th parallel 1° of longitude would be one-half of 69 miles. I was planning a trip from Wichita to Hot Springs, Arkansas, a couple of months ago, and I asked my class to figure the distance by this method considering all roads to run north and south or east and west. As you probably suspect, they obtained two answers differing by about ten miles, and both answers were correct! They discovered incidentally that it was shorter to go east and then south, than to go south, and then east. To find the distance along a great circle between two points, the sine-cosine formula derived from spherical trigonometry may be used. When the United States system of surveying the public lands was inaugurated, Congress required by law that "public lands shall be divided by north and south lines, run according to the true Meridian, and by others crossing them at right angles so as to form townships six miles square." The august body had forgotten that the distance between the Meridians varies as the cosine of the latitude.

There are four recognized types of air-navigation: piloting, dead-reckoning, radio-navigation, and celestial navigation.

Piloting consists of conducting an aircraft from one place to another by observation of land-marks either previously known or recognized from a map. Excellent maps for this purpose are published by the United States Coast and Geodetic Survey showing all land-marks which are most visible from the air. Before starting on the journey, the pilot calculates the distance to each land-mark along the route; the time at which he should arrive there; the amount of fuel needed for the trip, and must often calculate his time so as to reach his destination before dark.

Dead Reckoning is the method of determining the geographical position and maintaining the required course by applying the ground speed and direction made good over a certain period of time, from the point of the last known position. The pilot must determine his true course on each leg of his journey and correct for variation, deviation, and wind-drift in order to get his compass reading. All wind-drift problems can be solved by geometric constructions. Many mechanical devices are now on the market by which problems of dead-reckoning may be solved, but in order to use them intelligently the pilot must understand their mathematical make-up.

Radio Navigation is the method of conducting an air-craft by radio aids, such as the radio beacon, radio direction finder, and radioed bearings. It involves many geometric principles, such as intersections, bisectors, positive and negative angles, quadrants, etc. In fair weather it is possible to fly by this form of navigation alone, but adverse weather conditions interfere seriously with radio transmission, and it becomes necessary to resort to other forms of navigation.

Celestial Navigation is the method of determining the geographical position by means of observation of celestial objects, the sun, the moon, planets, and stars. The

essential equipment for celestial navigation is the chronometer, a sextant for measuring angles, and the *Nautical Almanac* for the year. The use of celestial navigation requires a knowledge of plane and spherical trigonometry and probably for that reason has been neglected by most pilots. Many crashes could have been avoided if the aviator had been familiar with this form of navigation. It is absolutely essential for flying over large bodies of water, and for stratosphere flying which is rapidly becoming prominent. Besides being very useful the study of celestial navigation is very fascinating and has become a hobby of many who are not themselves aviators.

Here I wish to make a plea for spherical trigonometry as a part of the high school course in mathematics. It abounds in fascinating problems, such as finding time by measuring the elevation of the sun at a certain date; obtaining true north and south line from the stars or sun; finding the time and direction of sunrise and sunset at various latitudes and the air distance from place to place by use of the sine-cosine formula etc. I have never been able to see why spherical trigonometry, which preceded plane trigonometry in point of time, receives so little attention in our mathematics courses.

The Weem's System of celestial navigation is becoming increasingly common because it is possible to get a fairly accurate "fix" within five minutes of time. The elevation angle of two celestial bodies is measured by the sextant and the sidereal time is noted. By means of ingenious graphs it is then possible to read directly the latitude and longitude to a surprising degree of accuracy.

Many may agree that navigation is a fine subject of study for boys but that it would be of little value to girls. However, we have good reasons to believe that girls will pilot airplanes as they now drive automobiles. I can remember when a woman was not considered capable of driving automobiles, and some people may still feel that way. Is it not possible that an airplane pilot would welcome a back-seat driver to figure his navigational problems? Mrs. Anne Lindbergh is a shining example of a woman who learned navigation as a practical hobby and did the back-seat driving for her husband across the Pacific, and also on their Atlantic tour.

The Red Cross

In the Fall of each year the Red Cross membership drive provides an excellent real-life situation for the use and appreciation of mathematics. Since it comes at a specified time the teacher can easily include it in the plans for the year, use it as an incentive or prepare for it well in advance if advisable. We all know the humanitarian values of this project but too often overlook the values for the teaching of mathematics and the opportunities for correlation and cooperation with other instructors and other subjects.

Junior High pupils may compute the percentage ratings of each group and make graphs or other pictorial representations to illustrate these. The intermediate grades may find the total contributions and work out individual and class averages. Even the youngest children might learn to identify the different coins as they are brought in and use them in simple problems or games. An assembly program which included the mathematical side of the "Red Cross Roll Call" would be valuable and prove enlightening to the entire school community.—Elizabeth B. Keep, Foxwood School, Flushing, Long Island.

On to Bethlehem!

All members of the National Council of Teachers of Mathematics should urge their friends to attend the Eighth December meeting of the Council at Bethlehem, Pa. on December 31 and January 1. Headquarters at Hotel Bethlehem. Make your reservations now.

The Problem of Ninth Year Mathematics

By Benjamin Braverman
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There are obvious reasons why ninth year mathematics has challenged and will continue to challenge the best thought of teachers of secondary mathematics.

One reason is that, of the various courses in secondary mathematics, ninth year mathematics has the largest pupil enrollment. This number might even be larger were it not for two facts. In the first place, many pupils, particularly in the 8–4 type of school organization, are barred from further study of mathematics in the secondary school by courses of study prepared by educators unsympathetic or definitely hostile to mathematics as an important element of the curriculum. In the second place, many pupils, particularly those who may have had some difficulty with mathematics in their first eight years of schooling, are shooed away from any further work in the subject by school advisers who fail to realize that these are the very pupils who could profit most by a new orientation to mathematics through the right kind of a high school course in it. It must be regretfully admitted, however, that such a course does not yet exist in many school systems, so that no fault can be found with school advisers for keeping these pupils out of traditional ninth year algebra.

But the large pupil enrollment in ninth year mathematics would not make the situation so difficult were it not for the great heterogeneity of this group. One expects, of course, a wide variation in native ability, as measured by the intelligence quotient, in a group as large as the one enrolled in ninth year mathematics. But, in addition to wide differences in ability, these pupils present equally wide differences in previous preparation, in habits of work, in attitudes toward learning, and in plans for the future.

Every teacher of ninth year mathematics has encountered at one time or another the most amazing individual examples on the part of his pupils, not only of serious deficiencies in the fundamental arithmetic skills and concepts, but of a complete lack of number sense. The most striking example of this in the writer's experience was the case of a boy of 15 at the High School of Commerce, who, in dividing 51 by 2, wrote as his answer, $210\frac{1}{2}$. When asked to explain this surprising result, the youngster said: "Two into 5 goes 2 times and 2 over. Two into 21 goes $10\frac{1}{2}$ times." But one does not have to cite individual cases of such serious deficiencies in fundamental mathematical skills and concepts to prove the point that there are extremely wide differences in the previous mathematical preparation of pupils exposed to ninth year mathematics. Several group studies have been made of pupil achievement in arithmetic skills and thought at the beginning of their study of ninth year mathematics. Thus, in a study of about 460 pupils made at the High School of Commerce by the writer about five years ago and reported in detail in The Mathematics Teacher for November, 1939, 75% were unable correctly to divide 216 by .18; the same per cent were unable correctly to multiply $2\frac{1}{2}$ by $2\frac{1}{2}$; and as many as 33% were unable to do the amazingly simple example of finding the product of 12 and 3.

But it is not only in native ability and previous preparation that pupils enrolled in ninth year mathematics show wide differences. In habits and attitudes and in general intellectual interests, there is as much, if not more, variation. A generation ago, the pupils in our public high schools, by and large, were there to satisfy a natural craving for knowledge or to fit themselves for a career requiring a higher education. Today that type of pupil is in a

very small minority. The raising of the compulsory school age, the reduction in employment due to the technological advances in industry and the prolonged economic depression, and the increased social and economic value placed upon college-trained men and women have brought to our public high schools and, hence, to ninth year mathematics many pupils with little or no intellectual interests and hence devoid of any inner urge for learning, with little or no pride in workmanship and hence lacking in neatness and orderly organization of written work, with little or no ideal for perfection and hence satisfied with mere completion rather than correctness of work assigned, with little or no habits of reflective thought and hence arriving at the most absurd conclusions without in any way being mentally disturbed, and finally with a general lack of ambition in and attention to matters involving learning and hence apparently unable to follow the simplest directions.

In plans for the future, too, there is great heterogeneity in the group at present studying ninth year mathematics. A generation ago, all pupils studying ninth year mathematics were definitely preparing for college and hence were going to pursue high school mathematics further. Today, in spite of the fact, as already pointed out, that many pupils are planning to go to college because of the increased social and economic value of a college education, it is safe to assert that only a minority of the enrollment in ninth year mathematics is going on to college. It is to be regretted that complete statistical information along this line is unavailable. However, in the High School of Commerce, of approximately 1,100 pupils taking ninth year mathematics during the term ending June, 1940, only about 250 were preparing to enter college. That fact is of the greatest importance. It means that for most pupils enrolled in ninth year mathematics, contact with it will cease at the end of that year, unless, as may be the case in a few schools, specialized courses in mathematics for the non-college pupil are offered.

What does the extreme heterogeneity of the pupils in ninth year mathematics mean in regard to the instructional material for that grade? To ask the question is to suggest its answer. There must be a differentiation of subject matter to meet the wide differences, as already pointed out, in native ability; previous preparation; habits, attitudes, and ideals; and plans for the future. One should not confuse the differentiation of instructional material suggested here with differentiation of problem material. A poorly prepared pupil, with an I. Q. of about 90, with hardly any intellectual interests, desirous of obtaining any kind of employment upon completing his high school course can be taught to divide $2x^3+21+5x-10x^2$ by $x-3$, but at what expenditure of time and labor and what a sacrifice of instructional material that would be so much more profitable to this type of pupil!

That the desirability of differentiated subject matter has been recognized by teachers all over the country is evidenced, not only by the many recent textbooks in ninth year mathematics, by the many new local courses of study for the ninth year, such as the terminal course prepared by the New York State Department of Education a few years ago, the course, still in the experimental stage, for the non-academically minded pupil sponsored by the teachers of mathematics in New York City, but by the latest recommendation of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics to be found in its report published last year for a course in general mathematics for the ninth year in place of the traditional ninth year algebra.

Most people are agreed that the course that will best meet the needs of the great majority of pupils in the ninth year and, at the same time, serve as the most effective learning medium is a course in general mathematics. There seems to be complete agreement too that the major ingredients

of this course should be some arithmetic, some algebra, including graphic representation, some geometry, and some trigonometry. However, there is considerable difference of opinion as to just what topics should be included under each of the major ingredients of the course, particularly the arithmetic and the algebra.

The serious deficiencies in fundamental arithmetic skills on the part of many pupils in ninth year mathematics, already pointed out, and their limited intellectual interests suggest at once that the arithmetic in the general mathematics course should contain a considerable amount of remedial work in fundamental skills and concepts, and a little consumer and social arithmetic. But, as Betz has warned, we must be careful not to carry the topic of social arithmetic to foolish extremes. The boy or girl coming from a home where uncertainty exists as to whence the money to pay for the next week's food or the next month's rent is to come is not going to be vitally interested in the problem of owning vs. renting a home, of the upkeep of a car, or in the merits of stocks and bonds as investments. Besides, the pupils in the ninth year are too immature to appreciate the more difficult topics of social arithmetic.

Also, the fact that only a small minority of ninth year pupils are planning to go to college and pursue the further study of mathematics suggests also that the geometry in the course in general mathematics should include a good deal of intuitive and constructional work with a brief introduction for the brighter pupils to demonstrative geometry. The same consideration, together with the desirability of the topic because of its concreteness, its emphasis upon functional thinking, and its ability to integrate so well the other ingredients of the course, also suggests that graphic representation play a very important part in this course in general mathematics.

But if the course in general mathematics is to serve its purpose and be generally adopted for all ninth year pupils who need it, it must contain more algebra than has been included in many of the recent courses in general mathematics, as represented by published texts. Otherwise, it runs the danger of becoming an emasculated course in mathematics intended only for the approximately 20% dull normal at the extreme left end of the normal frequency distribution. In order to have the course meet the needs of the large 60% of average ability in the middle of the normal frequency distribution and serve as a proper medium of instructional material for them, it must contain a suitable amount of the right kind of algebra.

The suggestion that algebra play as important a part as arithmetic, geometry, and graphic representation in a course in general mathematics for the average pupil in the ninth year has not been generally accepted. One reason may be our failure to put traditional ninth year algebra across to this type of pupil. Another reason may be our mistaken idea, due to the traditional presentation of the subject, that complete mastery of a large series of very difficult techniques in that subject must be achieved before the many applications of this subject to vital life situations can be considered.

In connection with algebraic techniques, is it not true that we have blindly accepted a great many of these techniques without any investigation as to whether they are really needed later, not in artificial, formal situations, but in real, significant life situations. Mention has already been made that the only time a pupil may use his knowledge of algebraic long division is in the solution of higher equations. The question that arises, therefore, in connection with this technique is why should it not be taught at the time it will be needed and to the very few who will need it, instead of in the ninth year to the great many who will never have any occasion to use it. A tremendous amount of time is still spent on factoring, but apart from the monomial factor case, are not all of the applications of the other cases entirely artificial? And

even the monomial factor case need not be taught as factoring, since any pupil who has mastered division by a monomial will understand what to do when the need for writing certain dividends as the product of a monomial divisor and the resulting quotient arises. We also fret and fume over fractions and fractional equations of extreme, formal types, becoming very discouraged because pupils cannot be taught to handle properly examples like $(4a-b)/3-(2a+b)/6$ or $4-5(y-3)/4=(y-13)/6$, forgetting that the pupil will profit more by an example like $\frac{1}{3}x+\frac{1}{4}x$ and that the solution of vital, practical, and significant problems from life situations will not require a fractional equation of greater difficulty than $C+\frac{2}{5}C=30$ or $12+\frac{1}{3}S=S$. Then we spend much time and labor on radicals in the mistaken idea that pupils who cannot combine $2\sqrt{12}+3\sqrt{27}$ into $13\sqrt{3}$ will not be able to handle properly the geometry of the right triangle. But has any one yet come across a significant application of the Pythagorean theorem suitable for ninth year pupils that requires the addition of two radicals? These illustrations of the futility of many algebraic techniques that are still taught and which unfortunately are still regarded by many teachers as the sine qua non of algebraic instructional material in the ninth year could be continued almost without end.

But, enough of destructive criticism. The cluttering up of algebra with so many techniques of questionable value has kept out of the course, not only the many worthwhile applications of the really essential techniques to problem situations, but the equally important applications of algebra as a mode of thought. These applications, with the techniques upon which they are based, should be the contribution of algebra to the course in general mathematics.

What are these worthwhile applications? Only two types of applications will be discussed here, those of algebraic language, including the formula, and those of the algebraic equation.

It really is amazing that so little use has been made of algebraic language to establish generalized relationships of interest and importance to the ninth year pupil. Thus, the difference between any two-digit number and the number with the digits reversed is always divisible by 9—a principle that has a most important practical application to bookkeepers and accountants. By the use of algebraic language, the universal truth of this principle can be established, since $(10a+b)-(10b+a)$ is $9a-9b$, and the latter expression is, of course, always divisible by 9. Similarly, the various tests for the divisibility of, numbers by 2, 3, 4, 5, etc., can be easily established for all cases by using the general number of the algebra. Also, is there any reason why the general truth of the principle that in any proportion the product of the means is equal to the product of the extremes should be postponed to the tenth year? If established in the ninth year, this principle offers a pupil the most convenient method to solve equations of the type, $a/x=b/c$, the type a pupil will encounter in all interpolations.

In connection with the formula, our traditional applications have been limited to translation and direct evaluation, with perhaps some functional thinking and graphic representation. It is well to point out here that in the direct evaluation of formulas, we have a splendid opportunity to do some very effective remedial work in arithemetic by emphasizing more than we have been doing examples of the type: By the use of the formula, $A=s^2$, find the area of a square whose side is $2\frac{1}{2}$ inches, or, by the use of the formula, $n=c/p$. find the number of pounds of sugar that can be bought for \$2.75 at $5\frac{1}{2}$ cents a pound. But, have we pointed out the value of the formula for planning and organizing the solution to a problem? Compare the accompanying solutions to the simple problem: Find the interest on \$1,750 at 3% for 6 months, one by the conscious use of the formula, the other without such conscious use.

(1) $p = 1750$ $i = prt$
$r = 3\%$ $= (1750)\ (.03)\ (\frac{1}{2})$
$t = 6$ months $= (875)\ (.03)$
$= 26.25$

(2) $p = 1750$

$r = 3\%$

$t = 6$ months

$$\begin{array}{r} 1750 \\ .03 \\ \hline 2\,/\,52.50 \\ \hline 26.25 \end{array}$$

But the formula has an even more important application. The derivation of certain formulas can be used to illustrate to the pupil the inductive method of thought. One wonders why, in our ninth year algebra, so far we have steered clear of deriving any of the formulas representing rules unfamiliar to the pupils. A mathematically rigorous derivation is, of course, not always possible and is not implied in the use of the word, *derivation*. A possible explanation for this situation may be our mistaken idea of holding the pupils responsible for the reproduction of everything presented to him. But we have abandoned that idea in demonstrative geometry, so why not in ninth year mathematics? Besides illustrating the inductive method of thought, the derivation of the formulas used will satisfy the adolescent's craving of the why and wherefore of things. The accompanying paragraphs indicate how a derivation of a formula may be effected for the ninth year pupil. The formula used is the formula for the last term of an arithmetic series. A conversation between two boys is used to arouse the pupil's interest and to encourage him to read the complete derivation for himself.

" 'The other day, Tom, I came across this interesting problem in an intelligence test conducted by a daily newspaper. A man agreed to dig a well 100 feet deep at the rate of 1 cent for the first foot, five cents for the second, nine cents for the third, and so on, increasing the price 4 cents every foot. How much did he receive for digging the last foot?'

'It sounds interesting enough, Bill. Can't we obtain the result by writing down the various quantities until we get the hundredth?'

'Of course, we can do it that way. But would you enjoy doing it that way, especially when mathematics offers us a much simpler way for doing it? Suppose we write down a few of the numbers that represent the cost of digging the well.'

'All right. The first is 1, the second is 5, the third is 9, the fourth is 13 . . . '

'Good. By the way, the succession of numbers you have just obtained, where one is obtained from the one just before it by adding the same number all the time is called an arithmetic series. Now, instead of writing them the way you wrote it, let's write them this way: the first is 1, the second is $1+4$, the third is $1+2(4)$, the fourth is $1+3(4)$. Each number is still 4 more than the preceding number.'

'Yes, I see that. But what have you gained by writing them your way?'

'Have you noticed the relation between the coefficient of 4 and the order of the number?'

'Isn't it one less than the order of the number?'

'Correct. Can you give me the tenth number, then?'

'Let's see. It will be 1 plus some number times 4. And that number will have to be one less than the order of the number. One less than 10 is 9. I have it, Bill. The tenth number is $1+9(4)$.'

'Splendid. And the fiftieth number?'

'$1+49(4)$.'

'Fine. And now, the hundredth number?'

'$1+99(4)$.'

'Excellent. Now let's simplify it.'

'Well, multiplication comes first. 99×4 is 396. $1+396$ is 397. Therefore, the cost of digging the hundredth foot is 397 cents or $3.97. I never realized that we could get the result without writing out all of the hundred numbers.'

'Another example of the power of mathematics. Now let's see if we cannot generalize what we have just done, so we

can work similar examples just as fast. Suppose our problem had been a trifle different and had said that the man received 6 cents for the second foot, 11 cents for the third, so that the price increased 5 cents instead of 4 cents, for every foot. What would have been the cost of the hundredth foot in that case? Remember that the cost of the first foot is still 1 cent.'

'Let's see. Before it was $1+99(4)$. Hence, it will now be $1+99(5)$.'

'Fine. And now, suppose the increase for each foot had been d cents.'

'The result would have been $1+99d$. By this time, I know, Bill, that there is no difference between the numbers 4 and 5 and the number d.'

'Splendid. Now let's introduce one more change in our problem. Suppose the cost of the first foot had been 10 cents instead of 1 cent and each foot thereafter d cents more. What would have been the cost of the hundredth foot?'

'Why, $10+99d$.'

'Good. And if the first foot had cost a cents?'

'$a+99d$.'

'Excellent. Now, suppose the well had been 150 feet deep. What would have been the cost of the 150th foot?'

'$a+149d$.'

'Of the 200th foot, assuming the well to be that deep?'

'$a+199d$.'

'Fine. And now, let's make it very difficult. Suppose the well had been n feet deep.'

'Let's see. In the other examples, I have had to take one less than the order of the number. The quantity that is one less than n is $n-1$. Hence, the expression would be $a+(n-1)d$.'

'Splendid. Now, if we call this expression l, we have the formula $l=a+(n-1)d$, which you will now find is a most convenient algebraic expression for finding any number in an arithmetic series quickly and easily. For instance, a boy joins a Christmas Club, depositing 10 cents the first week, 15 cents the second week, 20 cents the third week, and so on. How much will he have to deposit the fiftieth week?'

'Well, now, let's see. I have an arithmetic series because each number is obtained from the one before it by adding 5. The first term here, or a, is 10. The quantity being added to each number to get the next is 5, so the d in the formula is 5. The item we want is the 50th, so n is 50. Substituting these values in the formula, we have $l=10+(50-1)5$, or $l=10+49(5)$. Hence, $l=10+245$, or 255. Therefore, the fiftieth deposit will be \$2.55'."

In connection with the equation, one is simply amazed at the possibilities for worthwhile applications offered by the first type studied, the type, $ax=b$, with a an integer, a common fraction, or a decimal fraction. First, it might be well to point out that in the acquisition of the technique for the solution of this type of equation, we again have a splendid opportunity to do most effective work in remedial arithmetic by having the pupil solve equations like $.18p=216$, or $(5/8)p=1.50$. It is assumed, of course, that the pupil is taught the one general method for solving this type of equation, namely the division of both members of the equation by the coefficient of the unknown.

The first important type of application presented by the equation, $ax=b$, is in the indirect evaluation of such mensuration formulas as $A=lw$, $A=\frac{1}{2}bh$, $V=lwh$, $C=\pi D$, etc. Thus, by the use of the formula, $A=\frac{1}{2}bh$, find the altitude, h, of a triangle whose area, A, is 360 square feet and whose base, b, is 27 feet. Or, what should the height of a room be if it is to hold 7,500 cubic feet of air, and if its length is to be 30 feet and its width 20 feet. Finally, if a very difficult example is desired, a circular running track, 4 laps to a mile, is to be built. What should the diameter of this track be?

The second important type of application presented by the equation, $ax=b$, is the economy of rules and formulas made

possible by its use. Here is a distinct and significant advantage offered by the algebra which we do not exploit to the fullest degree. In arithmetic, we teach the pupil three rules for percentage. But, by means of the formula and the equation, these three rules can be replaced by the one formula, $p=br$. Thus, the solution to the problem—27 out of a class of 36 passed a certain test. What was the per cent passing?—reduces to the equation $27=36r$. Similarly, we give the pupil four rules for simple interest in arithmetic. But, by means of the formula and the equation, these rules can all be replaced by the one formula, $i=prt$. Hence, the solution to the problem—If Mr. Jones borrowed \$1,500 at 6% and paid back \$1,522.50, how long did Mr. Jones keep the loan—becomes the equation, $22.50=90t$. Even in our traditional algebra, we forget sometimes the economy of the equation and burden our pupils with the three formulas, $d=rt$, $t=d/r$, and $r=d/t$, when the one formula, $d=rt$, together with the equation, should solve any example dealing with uniform motion.

Finally, there is the verbal problem requiring complete analysis based on the equation $ax=b$. Here are a few illustrations of the kind of problems that can be used. They may be a little more wordy than the traditional problems that we have been using. They can also be solved arithmetically. But so can the traditional problems. However, would problems of this type require any defense of algebra?

"1. Jack's father is a salesman for the Excelsior Heating Co. At the present time, he is paid a weekly salary of \$54. But business in the heating line has been rather good lately and Jack's father is, therefore, seriously considering asking his company to place him on a commission basis. Salesmen in this line of business are paid a 5% commission. What Jack's father wants to know before making his decision is how much business he would have to produce a week in order to earn in commissions the same amount he is now earning as a salary.

"2. Mr. Jones, who is married and has two small children, was visited by an insurance agent the other day who urged him to take out additional life insurance. In order to reach an intelligent decision, Mr. Jones had to determine how much interest a year the money he would leave in insurance would be able to earn for his dependents, if invested conservatively. He estimates that his family would need a minimum of \$2,600 a year to support itself in comfort when he is gone. He also knows that the highest rate of interest that can be received in a safe investment at the present time is 3%. How much insurance should Mr. Jones carry in such a case?

"3. Very unfortunate circumstances compel Mr. Morgan, a victim of the usury racket, to borrow \$5 from one of these racketeers for a week. The racketeer lends him the money, but insists that he pay back \$6.00 at the end of the week. What rate of interest was the racketeer exacting?

"4. Jane paid \$1.50 for $\frac{5}{8}$ of a yard of cloth. She later needs another yard of the same material. What should she pay for that one yard, assuming that the price per yard has not changed?"

Verbal problems of the same concreteness and close connection to life exist for equations of the type, $a=b-x$, $ax+b=c$, $ax-b=c$, if we would only look around for them, instead of wasting so much time and energy to get pupils to solve monstrosities like: $x(2x+1)-3(4x+2)=2x^2-45$. The equation should be regarded as a means to problem solution and not as an end in itself. Thus, the cost formula from bookkeeping, $C=I_1+P-I_2$, together with the equation of the type, $a=b-x$, provides us with a vital and absolutely real problem like the following:

"Early in the morning of February 23, 1937, a ruinous fire broke out in the plant of the Reliable Shirt Co. Before it was checked, the entire stock of the company had been reduced to ashes. Fortunately, no lives were lost. Fortunately also for the company, it carried ample insurance and its books, which had been placed in a fire-

proof safe, had not been destroyed. In filing its claim with the insurance companies, the Reliable Shirt Co. had to establish the value of its inventory at the time of the fire. It couldn't do this by taking a physical inventory, that is, by examining each item of its merchandise and determining its cost. As has already been stated, its valuable stock of shirts was one mass of ashes mixed with water. Its books, however, showed that on January 1, 1937, it had an inventory of $36,750; that its sales from that date to the time of the fire were $144,800, and that its purchases, including factory labor for the same period, were $102,500. Also, an examination of the company's financial history over a five-year period prior to the fire showed that the cost of the goods sold by the company averaged 75% of the sales. From these facts, determine the value of the merchandise of the Reliable Shirt Co. at the time of the fire."

Does the problem sound a little too dramatic? Let's not forget that we are teaching boys and girls of 13, 14, and 15 who like the dramatic.

Mention has already been made that the vital applications of fractional equations do not require equations of greater technical difficulty than equations like $a/x=b/c$, $x\pm(a/b)x=c$, and $c+(a/b)x=x$, with a/b either a decimal or a common fraction. The first type of equation is a proportion and its solution, as already pointed out, should be based on the equality of the products of the means and extremes of a proportion. This type of equation is used not only in all interpolations, but in the countless real problems that the principle of proportion presents. Here is one a little different from the ordinary, but very real:

"You are taking a long trip and you are driving at an average speed of 40 miles an hour. Only about 15 or 20 minutes of daylight remain. A marker on the road says that the next town, Mesquite, is 12 miles distant. As a careful driver, you are averse to driving in the dark. You, therefore, want to know whether you will reach Mesquite before dark. How long is it going to take you to reach that town?"

Finally, here are some vital problems leading to the fractional equations already mentioned:

"1. A certain chain clothing store has a fixed selling price of $22.50 a suit. If it operates on a margin of profit of 40% on cost, what is the highest price it can afford to pay for its merchandise?

"2. You are the manager of the Full Value Department Store. Among your numerous duties are the following: to examine the merchandise received, to check the quantity and price against the invoice, to place a sales tag on the merchandise, and then send it down to the selling floor. A suit of clothes has just come in from the Rite Fit Garment Co. The cost of it is $15.75. What selling price should you place on it? The executive department has instructed you, earlier in the year, that in fixing selling prices, you are to use a 30% margin on the selling price.

"3. The profits of the Pioneer Dress Corp., after deducting all necessary and business expenses, amounted to $13,500. As a reward for the excellent managerial ability of Mr. Henry Green, the general manager of the company, the directors voted him a bonus of 12½% of the net profits remaining to the corporation after deducting this bonus. What was Mr. Green's bonus?"

These, then, are the general principles upon which an effective course in general mathematics for the ninth year, embodying arithmetic, algebra, graphic representation, geometry, and trigonometry, should be built.

For American Education Week

Urging Nation-wide observance of American Education Week, November 9–15, John W. Studebaker, U. S. Commissioner of Education, today announced publication of two U. S. Office of Education publications—timely aids for use by teachers, speakers, and writers during this special week.

The bulletins are "Voices of Democracy," and "Expressions on Education by Builders of American Democracy." They quote all U. S. Presidents and many philosophers, statesmen, writers, and others on education, liberty, and democracy.

The Mathematics Vocabulary of Current Periodical Literature

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IN RECENT years educators have placed more and more stress upon the topic of vocabulary comprehension as a factor in determining the degree of success of pupils in various branches of school work which they undertake.

Brooks in quoting a study made by Roling, Blune, and Morehart states:[1] "Pupils often fail to get the meaning of a problem because of word difficulties, . . . inability to interpret the meaning of words used. Such words as *average, area, rate, ratio, product,* were some of the words for which we found very indefinite meanings in the minds of pupils.

Vocabulary difficulties . . . are real and are always present, they hinder the pupils in their understanding of the full import of the problems to be solved. The words and phrases used are often strictly mathematical and bear a sense peculiar to the subject. When pupils fail to interpret them correctly, they become a source of difficulty."

These same reading difficulties arise not only in connection with school work but in all types of reading material that the pupil may come in contact with.

It was the purpose of this paper to make a study of current periodical literature with a view of attempting to gain some insight into the mathematical vocabulary necessary for intelligently reading such literature.

A criterion for selecting mathematical terms was established which provided for selecting all terms that dealt with magnitudes, quantities, and numbers; operations with numbers; properties of form and space; units of measure; names of persons involved in mathematical operations; expressions that could be expressed by means of a formula or equation; and prefixes of a strictly mathematical nature.

TABLE I

Mathematical Terms Found Which Occurred Ten Or More Times

Mathematical Term	F	Mathematical Term	F
percent	184	decade	19
hour	127	less	19
million	108	sum	19
mile	102	thirds	19
average	87	annual	18
thousand	73	expenditure	18
acre	72	stock	17
foot	68	check	16
debt	61	parallel	15
tax	57	statistics	15
cent	54	corporation	14
increase	54	fourths	14
dollar	51	angle	13
pound	49	area	13
number	47	dividend	13
half	45	value	13
population	44	weight	13
dozen	41	yard	13
minute	41	bushel	12
century	36	degree	12
bank	34	divide	12
per	34	formula	12
profit	34	minimum	12
income	32	plus	12
loan	32	quarter	12
total	32	banking	11
estimate	30	invest	11
figure	30	miles per hour	11
rate	30	budget	10
amount	29	double	10
billion	28	interest	10
inch	26	percentage	10
hundred	25	salary	10
ton	25	score	10
approximate	22	seconds	10
reduce	20	share	10
add	19	temperature	10
capital	19		

In order to obtain a variety of articles taken from several different magazines it was decided to use six consecutive issues of the magazine, *Readers Digest,* as a basis for the study. The writer did this

[1] S. S. Brooks, "A Study of the Technical and Semi-Technical Vocabulary of Arithmetic," *Educational Research Bulletin,* Ohio State University, V (May, 1926), 219–222.

knowing that the articles used in this periodical are condensed and hence would yield different results than would a study made from the original sources.

The six issues of the magazine were read in their entirety and all mathematical terms were underlined. These terms were checked by a second reader to reduce the subjective factor of human judgment to a minimum. The six issues of the magazine comprised a total of 340,185 running words. From these a total of 3,130 mathematical terms were found. However these 3,130 mathematical terms were made up of only 360 different mathematical terms.

The terms selected were tabulated according to frequencies of occurrence and then listed in descending order of their frequency of occurrence. Table I, shows those terms which occurred ten or more times in the six issues of the magazine.

In addition to the 75 terms listed above there were 285 terms which occurred less than ten times in the six magazines. Among these were such commonly used terms as *gallon*—9 times, *fraction*——7 times, *calorie*—6 times, *knot*—6, *board feet*—5, *lb.*—4, *broker*—3, *cylinder*—3, *square mile*—3, *axis*—2, *calculus*—2, *net profit*—2, *symmetry*—2, and *cubic inch*, *direct ratio*, *kilocycle*, *millimeter*, *prism*, *pyramid*, *sector*, etc., one time each.

Table II, summarizes the frequency of occurrence of the 360 different mathematical terms found with the percentage of the total number of terms found.

TABLE II

Frequency of Occurrence to the 360 Mathematical Terms

Frequency of Occurrence	Number of Terms	Percentage of Total Number of Terms
Once	133	37.0
Twice	46	12.8
3 to 5 times	72	20.0
6 to 11 times	43	11.9
11 to 20 times	31	8.6
21 or more times	35	9.7
Total	360	100.0

From Table II we see that 133, or 37 per cent, of the total number of different mathematical terms occurred but once in the six magazines examined, while but 35 different terms, or 9.7 per cent of the total number, occurred more than 20 times in the six magazines.

The 360 different mathematical terms were then checked with the Dolch "Grade Vocabularies"[2] to determine the school grade classification of each word found. Table III shows the total number of terms found at each grade level with their percentage of the total, and the total number of different mathematical terms found at

[2] E. W. Dolch, "Grade Vocabularies," *Journal of Educational Research*, XVI (June, 1927), 16–26.

TABLE III

The Number of Mathematical Terms Found in the Six Magazines Which Appeared in Each Grade Level of the Dolch "Grade Vocabularies"

Grade Level	Total No. of Terms	Percentage of Total No. of Terms	Total No. of Different Terms	Percentage of Total No. of Different Terms
Grade I	748	23.9	36	10.0
Grade II	780	24.9	50	13.9
Grade III	270	8.6	21	5.8
Grade IV	386	12.3	35	9.7
Grade V	413	13.2	28	7.8
Grade VI	96	3.1	34	9.4
Grade VII	114	3.7	29	8.1
Grade VIII	67	2.1	21	5.8
Unclassified	256	8.2	106	29.5
Total	3130	100.0	360	100.0

each grade level with their percentages.

From Table III we are able to discern that the largest number of terms found are classified as being of either first or second grade level. It is also evident that the number of mathematical terms per grade declined quite sharply in Grades VI, VII, and VIII. This decline is due in part to the fact that many mathematical terms—such as *table, knot, score,* etc.—are listed in the Grade Vocabularies with their homographs which are used much earlier in life by children than are the mathematical terms.

Unfortunately the Dolch list does not classify words above Grade VIII; however, we may assume that many of the 106 terms found in the unclassified list—*graph, gross income, hexameter, income tax, kilocycle, loan value, margin of profit, mortality rate, net profit, odds, par, per capita, quantum, sector, segment, statistician, submarginal, symmetry,* etc.—are terms which are used in our high school courses of general arithmetic, algebra, or geometry.

There were 53 different units of measure in the list of terms found. Of the 53 units of measure, 19 occurred more than ten times (see Table IV), 6 occurred more than 50 times, and but two terms—*hour* and *mile*—occurred more than 100 times. These 53 units of measure occurred a total of 859 times in the six magazines.

In the light of the above findings it would appear that the following conclusions are justifiable:

1. Even though only 1 per cent of the words found in the six issues of the magazine were mathematical terms, it would not be possible to read intelligently the articles contained in these magazines unless the reader possessed a definite meaning of these same mathematical terms.

2. The teaching of mathematical vocabulary is a problem that all teachers in our schools, first grade through high school, must contend with.

TABLE IV

The Units of Measure Found in the Six Issues of the Magazine

Unit of Measure	F	Unit of Measure	F
hour	127	barrel	3
mile	102	fortnight	3
acre	72	penny	3
foot	68	square mile	3
cent	54	franc	2
dollar	51	horsepower	2
pound	49	nickel	2
dozen	41	pace	2
minute	41	peck	2
century	36	pint	2
inch	26	board foot	1
ton	25	candlepower	1
decade	19	centenary	1
yard	13	cubic inch	1
bushel	12	dime	1
degree	12	hexameter	1
miles per hour	11	kilocycle	1
seconds	10	man hour	1
score	10	mark	1
gallon	9	millimeter	1
calorie	6	per diem	1
knot	6	sen	1
meter	4	shilling	1
quart	4	square inch	1
square foot	4	square yard	1
ounce	4	tonnage	1
acreage	3		

3. Teachers in our schools should study materials appearing in magazines, newspapers, etc., so as to be able to determine better the technical vocabularies that we should teach in order to enable our pupils to reach such literature intelligently.

4. Efforts should be made to build up definite grade vocabularies in mathematics so that teachers may be able to determine with some degree of accuracy the vocabulary to be mastered in the particular grade levels in which they teach.

I HOLD every man a debtor to his profession; from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves, by way of amends, to be a help and an ornament thereunto.—*Bacon (Preface to "Maxims of the Law.")*

◆ THE ART OF TEACHING ◆

The Cross-Staff

By W. E. SCHIMPFF, *Technical High School*
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THE CROSS-STAFF and its application to surveying and navigation was used in earliest times. Apparently it was completely forgotten during the middle ages, but at about the time of the discovery of America it was revived, and during the period of great discoveries that followed the voyages of Columbus, it was the principal navigating instrument of those sailors who have left their wake on the history of the seven seas. There is no mention of the instrument by Christopher Columbus, but there is a vast amount of reference to its use by subsequent discoverers.

In its simplest form the cross-staff consisted of two bars of wood. The longer rod was called the staff. One end of the staff was held against the cheekbone of the user when making an observation. The short rod was called the cross. This rod could be moved at right angles along the staff, and was set at such a position on the staff so that two points could be observed at the extremities of the cross.

Its use was wider than the use of the astrolabe, which was employed for the most part in the measurement of vertical angles. In preference to the astrolabe the cross-staff was used in measurements of vertical angles up to sixty degrees above the horizon.

Newly discovered islands, headlands, rivers and bays were charted by the use of the cross-staff, and while we today would not regard the data obtained from cross-staff observations as very accurate, let it be definitely understood that the data from this instrument in the hands of a competent observer was good enough to enable subsequent voyagers to relocate the discoveries.

So closely did the settings on the cross-staff approximate the trigonometric functions that its use was readily understood by mathematically trained seamen. Many of the problems in the spherical trigonometry needed by the navigator and the surveyor could be solved by using the data secured with the cross-staff. The angular distance between two heavenly bodies, especially at night was just as easily taken as the vertical distance of the sun above the horizon in the day. Its use was not restricted as was the use of the astrolabe to vertical angles, and in consequence it was the far more useful instrument. The extremely difficult problem of finding the longitude of the place of the observer could be solved at that time only by cross-staff observations.

In additon to its use as a navigating instrument, the cross-staff was used by surveyors in the measurement of plane angles and the solution of many practical problems. The following problems are listed in the *First Book of the Cross-Staff*, published by Edmund Gunther in London, in 1673:

1. To find a height at one station by knowing the distance.
2. To find an height by knowing some part of the same height.
3. To find an height at two stations by knowing the difference of the same stations.
4. To find a difference by knowing a height.
5. To find a distance by knowing part of a height.
6. To find a distance at two stations by knowing the difference of the same stations.
7. To find a breadth by knowing the distance perpendicular to the breadth.
8. To find the breadth of two stations in a line perpendicular to the breadth, by knowing the difference of the stations.

The problems are, in the main, relatively simple. In the first problem we are asked to find the fourth term d of the proportion $a:b=c:d$, where a is the reading on the staff, b is the reading on the cross, c is the distance of the object and d the height of the object.

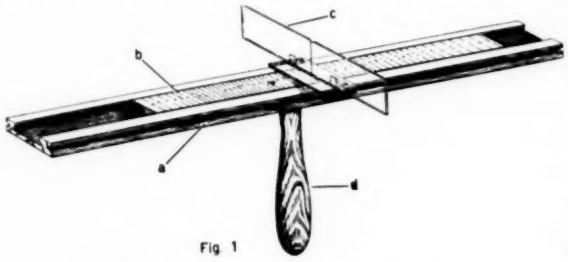

Fig. 1

Cross-staff designed by W. E. Schimpff: a, base or staff, b. slide with special scale, c. transparent cross member or vane, d. handle.

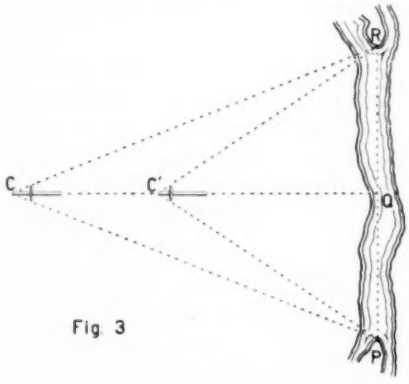

Fig. 3

Finding the distance between two inaccessible objects with the cross-staff—making observations at C and C'.

In the last problem the true genius of the mathematicians of that day is clearly demonstrated. In this problem, the only measurement made is made along a line perpendicular to the line whose length is desired, and the readings made on the cross-staff are made on the staff portion of the instrument, which staff is held perpendicular to the line to be measured also.

The sides of the cross-staff, which was always square gave four sets of graduations to the inventively inclined constructor of such an instrument. One side of the cross, as well as one side of the staff was marked in degress for the measurement of angles. Another side generally carried graduations up to 100 parts on both the cross and the staff. These graduations were in steps of 25, 50, 75, etc. As many staffs were three or four feet in length, interpolation was readily accomplished. Of the two remaining sides various graduations were recommended. These graduations depended upon the genius and fancy of the early surveyors.

While excellent descriptions of the cross-staff exist, there is no known specimen of the cross-staff to be found in a museum or in the hands of a private collector. It was a very good instrument while it was used, but promptly gave way to the sextant when the sextant made its appearance. The sextant was lighter, more easily used, and more accurate. But because of the service rendered by the early navigators of the period known as modern history, the cross-staff deserves a high place in the Hall of Fame of surveying and navigating instruments.

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The National Council of Teachers of Mathematics Official Notice

As SECRETARY of the National Council of Teachers of Mathematics, I officially announce the annual election of certain officers of the National Council, said election to take place at San Francisco, California, February 21, 1942. Article III Section 7 of the By-laws states: "At least two months before the date of the annual meeting, all members shall be given the opportunity, through announcement in the official journal, to suggest by mail for the guidance of the directors a candidate for each elective office for the ensuing year. At least one month before the annual meeting the secretary of the board of directors shall send to each member an official ballot giving the names of two candidates for each office to be filled. These candidates shall be selected by a nominating committee of the board of which the secretary shall be chairman. The election shall be by mail or in person and shall close on the date of the annual meeting."

At the Atlantic City Meeting, February 26, 1938, of the National Council the nominating committee consisting of the two most recent ex-presidents and the secretary as chairman (for this year: Martha Hildebrandt, H. C. Christofferson, and Edwin W. Schreiber), was instructed to prepare an official ballot suggesting two eligible candidates for each elective office and reserve a blank space for a third prospective candidate which may be written in by the voter. The officers to be elected at the 1942 San Francisco Meeting are: President, 1942–1943; Second Vice-President, 1942–1943; and three Directors, 1942–1944. Since there is to be no primary ballot, affiliated organizations, local clubs, or individual members who wish to present names for the official ballot should do so by writing the secretary immediately. The official ballot will be sent to members through the mail the first week in January.

The periods of service of the officers of the National Council, from its organization in 1920 to the present time, are printed on page 323.

EDWIN W. SCHREIBER, *Secretary*

L'Algebroi

(With apologies to Kipling)

When the last algebra problem is finished
And our brains are tired and dull,
When the oldest student is shirking
And the youngest, too, has a lull;
We shall rest,
And, overworked, we shall need it—
Sleep a million semesters or two,
Till the Master of all problem solvers
Shall set us a-factoring anew.

And they who had A's shall be happy:
They shall lord it over the rest,
They shall work at a ten-league blackboard,
With crayon the very best;
They shall have real authors for teachers,
Burnside-Graham and Hall:
They shall change all the signs in the text book
And never get mixed up at all.

And only the Master shall check them,
And only the Master shall blame;
And no one shall figure for money,
And no one shall figure for fame,
But each for the joy of his algebra,
And each, on the point of a star,
Shall give out results as he sees them,
And not copy his neighbor "over thar."—

IRMA HINRICHS
College of Saint Teresa
Winona, Minnesota

The National Council of Teachers of Mathematics

Organized 1920—Incorporated 1928

Periods of Service of the Officers of the National Council

Honorary Presidents

*H. E. Slaught, Chicago, Ill., 1936–1937

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1. C. M. Austin, Oak Park, Ill., 1920
2. J. H. Minnich, Philadelphia, Pa., 1921–1923
3. Raleigh Schorling, Ann Arbor, Mich., 1924–1925
4. Marie Gugle, Columbus, Ohio, 1926–1927
5. Harry C. Barber, Exeter, N. H., 1928–1929
6. John P. Everett, Kalamazoo, Mich., 1930–1931
7. William Betz, Rochester, N. Y., 1932–1933
8. J. O. Hassler, Norman, Okla., 1934–1935
9. Martha Hildebrandt, Maywood, Ill., 1936–1937
10. H. C. Christofferson, Oxford, Ohio, 1938–1939
11. Mary A. Potter, Racine, Wis., 1940–1941

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3. Eula Weeks, St. Louis, Mo., 1922
4. Mable Sykes, Chicago, Ill., 1923
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6. Winnie Daley, New Orleans, La., 1925
7. W. W. Hart, Madison, Wis., 1926
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9. Mary S. Sabin, Denver, Colo., 1928–1929
10. Hallie S. Poole, Buffalo, N. Y., 1929–1930
11. W. S. Schlauch, New York City, 1930–1931
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13. Mary A. Potter, Racine, Wis., 1932–1933
14. Ralph Beatley, Cambridge, Mass., 1933–1934
15. Allen R. Congdon, Lincoln, Neb., 1934–1935
16. Florence Brooks Miller, Shaker Heights, Ohio, 1935–1936
17. Mary Kelly, Wichita, Kansas, 1936–1937
18. John T. Johnson, Chicago, Ill., 1937–1938
19. Ruth Lane, Iowa City, Iowa, 1938–1939
20. E. R. Breslich, Chicago, 1939–1940
21. F. L. Wren, Nashville, Tenn., 1940–1941
22. R. L. Morton, Athens, Ohio, 1941–1942

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*Frank C. Touton, Los Angeles, Cal, 1926–1928
Vera Sanford, New York City, 1927–1928
William Betz, Rochester, N. Y., 1927–1929, 1930–1931, 1934–1936, 1937–1939
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Edwin W. Schreiber, Ann Arbor, Mich., 1928–1929
Elizabeth Dice, Dallas, Tex., 1928, 1929–1931
J. O. Hassler, Norman, Okla., 1928, 1929–1931, 1933, 1941–1943
John R. Clark, New York City, 1929–1931
Mary S. Sabin, Denver, Colo., 1929–1930, 1931–1933
J. A. Foberg, California, Pa., 1929
C. Louis Thiele, Detroit, Mich., 1931–1933
Mary Kelly, Wichita, Kan., 1932
John P. Everett, Kalamazoo, Mich., 1932–1934
Elsie P. Johnson, Oak Park, Ill., 1932–1934
Raleigh Schorling, Ann Arbor, Mich., 1932–1934
W. S. Schlauch, New York City, 1933–1935
H. C. Christofferson, Oxford, Ohio, 1934–1936, 1937–1939
Edith Woolsey, Minneapolis, Minn., 1934–1936, 1937–1939
Martha Hildebrandt, Maywood, Ill., 1934–1935
M. L. Hartung, Madison, Wis., 1935–1937, 1938–1940
Mary A. Potter, Racine, Wis., 1935–1937
Rolland R. Smith, Springfield, Mass., 1935–1937, 1938–1940
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L. D. Haertter, Clayton, Mo., 1936–1938
Virgil S. Mallory, Montclair, N. J., 1936–1938, 1939–1941
Kate Bell, Spokane, Wash., 1938–1940, 1941–1943
A. Brown Miller, Cleveland, Ohio, 1939–1941
Dorothy Wheeler, Hartford, Conn., 1939–1941
Hildegarde Beck, Detroit, Mich., 1940–1942
H. C. Charlesworth, Denver, Colo., 1940–1942
L. H. Whitcraft, Muncie, Ind., 1941–1943

* Deceased

EDITORIALS

How Important Is the Study of Mathematics?

THERE are so many articles and comments these days *pro* and *con* on the importance of the study of mathematics that one is at a loss to select the most useful and interesting ones to pass on to the readers of THE MATHEMATICS TEACHER, most of whom may not have read all of the articles concerned.

A recent editorial in *The New York Times* brought forth a reply which is quoted in full below:

MATHEMATICS HELD IMPORTANT

Proposal to Substitute History Units Regarded as Impracticable

TO THE EDITOR OF THE NEW YORK TIMES:

In your editorial "Appeal to the Colleges" you suggest that the college entrance requirements be modified by the exchange of two units of algebra for an equal number of American history. While I am in full accord with the President's plea for redoubled efforts by our schools and colleges in support of our democratic way of life, yet I do not see how these are to be made by disregarding the fundamentals of secondary education.

It is clear that American history is an essential ingredient in any prescription for cultural defense against nazism, but there is also a material side which seems to have been overlooked. We must have well-trained engineers, chemists and other technologists. In educating these men two years of high school algebra is an absolute and basic minimum. To require them to get this in college would delay their work for one year and render futile the present effort toward a three-year technological degree.

For these men—and they include an increasingly large part of the student body—the algebra-history exchange would be dangerous in the extreme. I do not think it far-fetched to consider the possibility that our present secondary school students will be playing a role in defense work before the end of the present conflict. For the future we must have on hand an abundant supply of economists, agrarian statisticians and their like to help reassemble the pieces of the jig-saw puzzle Hitler is making of our world. For these men also, if they are to do more than feed figures into formulas whose meaning they learn by rote, two years of algebra in high school is only a foundation for more mathematics in college.

Your list of minimum entrance requirements includes only eleven out of a possible twenty units, and it is manifest that the addition of two units of American history would, in the interests of the present emergency, cause no great difficulty. There would then be left seven units for "elegancies and superfluities." There are also degree requirements, and I would say that American history is of a parity with physical education, two years of which is required, and without degree credit, by most colleges.

While there is much to be said against regarding elementary mathematics as a frill on the academic costume, this is better left unsaid for the present.

A. D. WALLACE

Princeton, N. J. Aug. 15, 1941

The above letter stimulated a second letter to the Editor of *The Times* which ran as follows:

PHYSICISTS NEEDED

Shortage Laid to Failure to Major in Mathematics

TO THE EDITOR OF THE NEW YORK TIMES:

Writing in your Aug. 16 issue, A. D. Wallace emphasizes the importance of mathematics in general and of algebra in particular as a preparation in supporting our way of life. This point cannot be too strongly emphasized.

At present there exists a shortage of physicists in this country as well as in England. When one considers that fewer than 150 individuals enter the field of physics each year to make it a life profession, one realizes that in an emergency like the present we are severely handicapped in providing enough physics teachers for our educational institutions and at the same time furnishing a large group of those qualified to do basic research in physics for developing and improving the various gadgets so essential in modern, technical warfare.

I am convinced from many years of trying to interest young men in the field of physics that their objection to taking it up as a profession is due to their lack of training in mathematics.

Deterred by Calculus

The past Spring when 850 young men were scanning the college catalogue to find a suitable number of courses for the academic year immediately ahead of us, at least six said to me: "If I didn't have to take calculus, I would take advanced courses in physics." Mathematics beyond trigonometry was a hurdle which they refused to take. Algebra is unique in that it is one field of mathematics used in all other branches of mathematics and unless one can handle algebraic transformations the advanced courses are dull, uninteresting and stupid.

When students come to college unable to do the simplest kinds of algebraic transformations I believe it behooves the colleges to give special courses for such students. Of what use are advanced courses without the ability to handle the fundamentals? Only continued and extensive drill can make one expert in the handling of mathematics.

The refusal to take mathematics in college and learn to use it is an almost irretrievable loss, because few will come back after college days to learn it, whereas the reading of history, for example, may be done on vacation or any other time. The technique of mathematics is not usually acquired in that way.

Having acquired manipulative skill in handling mathematics, there is still the laborious and painful task of learning how to apply it to the problems of physics and engineering. Knowledge of mathematics does not necessarily indicate ability to use it in a practical way for the solution of engineering problems.

Application Important

There has just come to my desk a letter from a leading engineer in a large industrial research laboratory in which he found he was unable to cope with certain theoretical problems although he had been trained in advanced mathematics which included calculus, vector analysis, elliptic integrals, complex variables and modern theories of differential equations. The trouble as he saw it was that the courses were given by mathematical instructors who placed too much emphasis on manipulations and neglected or ignored the applications to practical problems.

As time was a factor he called in three Professors of Mathematics who failed to do what he wanted, but a fourth man who could see the correspondence between a physical situation and its mathematical form did succeed.

The road to having ability to use mathematics and apply it to the problems of physics, engineering, chemistry and other technical fields is an exacting one, but a challenge to those who have the stamina to tackle it.

Practical methematics must be taught in our high and preparatory schools and above all must be given a unique place in our college curricula. Our college courses in this field must teach our students how to apply their mathematics to the problems which are continually arising in this our technical world.

S. R. Williams

Amherst, Mass., Aug. 21, 1941

A final editorial appeared in *The Times* (August 26, 1941) which doubtless resulted from the previous editorial and letters will be of interest to all of our readers. It runs as follows:

The Mathematical Tool

There can be a bottleneck in mathematics just as surely as in machine tools. And to avoid it requires just as careful a program of long-range preparation. The War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America is right in proposing that grade-school boys of 11 and 12 who have mathematical aptitude be sought out and encouraged to take courses that will lay the foundation for careers in engineering, physics or directly in the technical branches of the armed services.

The kind of training here needed is not the kind emphasized in certain New York City high schools which make such a fetish of accurate arithmetic that 100 per cent is the only passing grade. Such a system may turn out some trustworthy bookkeepers, but it will not develop men with mathematical imagination, men who can use mathematics as a tool to shape out the approximations and compromises that are actually involved in engineering design.

The youthful mind, naturally imaginative, is riper than most pedagogues seem to think for symbols and instruments that offer interesting short-cuts through the drudgery of the multiplication table. The accountant doesn't add or multiply by hand—he leaves it to the calculating machine. The engineer doesn't fill a page with figures to extract a square root—he reads it from a slide-rule.

Thorton C. Fry, Mathematical Research Director of the Bell Telephone Laboratories, estimates in a new survey of industrial mathematics that in the design of a modern four-engined transport plane about 100,000 hours are spent on mathematical analysis of structures, performance, lift distribution and stability. In calculations of such magnitude short-cuts and approximations are worth money—they may save thousands of hours. Such work, and similar calculations called for in electric power, com-

munications and petroleum prospecting, must be directed by men who have got beyond the bugbears of arithmetic and are not afraid to look a differential equation in the face.

Probably a majority of the people are afraid of mathematics, and that awe is largely traceable to the early, teacher-inspired worry about not getting the example exactly right. Mathematics could be made a fascinating game. The pupil should be encouraged to approximate, or guess, the answer the minute the problem is put before him; then work it out the regular way and see how close he came; and finally check it with a slide-rule or calculating machine. That—skipping the middle stage of all-out arithmetic—is the way the engineer does it.

It is obvious that there is a revival of interest in mathematical education among those who think seriously about the matter. The one thing that teachers of mathematics need to remember is that they must continue to improve their organization of material and methods of teaching so that gain to all concerned will result.

W.D.R.

Mary Kelly

THE MATHEMATICS TEACHER regrets to announce the death of Mary Kelly who for many years has been a very energetic member of the National Council of Teachers of mathematics and who only recently served as vice president of that organization. An editorial in the January (1941) issue of THE MATHEMATICS TEACHER told of her recent retirement from active service in the Wichita (Kansas) schools.

The following tribute adapted from one prepared by her friends, Leida H. Mills and Gertrude Jackson Stice, which appeared in *Association Activities*, a publication of the Wichita City Teachers Association for September 1941, will be of interest to all of Miss Kelly's host of friends and particularly to those of us who knew her best:

Mary Kelly, respected and loved teacher in the Wichita Public Schools for many years, after one brief year of retirement from her arduous duties as Head of Mathematics in East High School, passed on Sunday, August tenth, 1941 "acrost the bar, into the boundless deep, home."

She leaves a devoted brother, John C. Kelly, and his wife, of 237 N. Crestway, a great niece, Peggy Davis, and many friends and students to mourn her passing.

Miss Kelly was a native Kansan, born in Leavenworth in 1870. When five years old she came with her parents to Wichita. After graduating from the Wichita High School, she entered the City Normal School under the direction of Miss Mary E. Rowe, and the next year began her long service in the Wichita Public Schools: First in the Primary Grade, then as Principal of Irving School before becoming Head of Mathematics in the High School.

She was an exceptional teacher in her chosen subject as many of her students, who entered various colleges, West Point and Annapolis, have been proud to acknowledge.

Her preparation was thorough and always progressive. She had an A.B. Degree from Friends' University, and her Master's from Columbia University, New York.

She served ten years on the Editorial Staff of the *Kansas State Mathematics Bulletin*.

As President of the Kansas State Association, and Chairman of the Wichita Sectional Round Table, she was instrumental in bringing men of national reputation to the state meetings. She was an honorary member of the National Honorary Education Society, Delta Kappa Gamma.

Miss Kelly, collaborating with Miss Weimer, now Mathematics Head in North High, and other teachers in her department, in 1928 compiled a *Syllabus and Exercises in Geometry*. This work, representing many hours of strenuous labor, is used as a supplemental text book.

Miss Kelly's fine library on mathematical subjects, including some priceless old books of Euclid, was given in her will to Friends' University.

A volume could be written upon her classroom work, her interest in the individual pupil, her helpful supervision and loyalty to her assistants. Her teaching, while thorough to the last degree, also emphasized character development as the main objective of education. She was always interested in the careers of her pupils and they repaid her with loyal affection.

Mary Kelly had an unusual aptitude, almost genius for hospitality and friendship. She loved to entertain her friends and she was an

ideal hostess, modest, with a quiet sense of humor and an enveloping friendliness.

Words are inadequate in this brief article to do justice to the life and character of Mary Kelly; but many, both here and in distant places will say, "She was my friend, loyal and true,—I shall miss her."

W. D. R.

How One "Key Man" Does It

THE following letter from a "Key Man" who for the good of the cause has taken a great interest in the work of The National Council of Teachers of Mathematics is one of best of several that we have seen that have been sent out in various places all over the country in an attempt to secure new members for the Council.

Rapid City, South Dakota
September 9, 1941

Dear Teacher of Mathematics:

Do you know: That there are 150 teachers in South Dakota who teach mathematics exclusively, and that only twenty of these 150 are members of The National Council of Teachers of Mathematics?

Do you know: That a subscription of THE MATHEMATICS TEACHER, the only magazine in America dealing exclusively with the teaching of mathematics in elementary and secondary schools, automatically makes you a member of the Council?

Do you know: That a subscription to THE MATHEMATICS TEACHER is only $2.00?

Do you know: That the address for THE MATHEMATICS TEACHER is 525 West 120th Street, New York, N. Y.?

Why Become a Member:

1. To increase your professional standing.
2. To become acquainted with the problems of teaching mathematics in our public schools.
3. To become acquainted with the new books that are ever being published in the field of mathematics.
4. To become acquainted with the leading mathematicians in America as well as successful classroom teachers.
5. To support the only National Organization which is continuously bringing to the attention of the educational world the necessity of teaching mathematics in our public schools.

Will you please return the enclosed postcard by October 6?

Sincerely yours,
Florence Krieger
"Key Man for the Western District"
Rapid City High School

THE MATHEMATICS TEACHER appreciates all of this type of pioneering that is being done by members of the Council all over the country.

W. D. R.

A Reaction of a Reactionary

(In Reply to A Student's Question.)

O tempora, o mores, she
Asks "Why should we study geometry?"
Enough to make a dozen Thales
Take up the cudgels with shillalahs,
A score or more Pythagorases
Cry "Build us another bridge for asses!"
And send full seven times seven Sages
Into as many separate rages.

One wonders what would wise old Plato
A maid so lost to reason say to;
If Euclid in his grave, this minute
Obliged to turn right over in it,
Will, roused from antique dreams, but mutter,
"Behold O great Athene th'utter
Folly of casting wisdom's pearls
Before these ultra-modern girls!"—

ANNE YOUNG

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
The Bronx High School of Science, New York City

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May, 1941, Vol. 48, No. 5.

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3. O'Toole, A. L., "College Geometry for Secondary School Teachers," pp. 319–326.
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NEWS NOTES

Defense Series Pamphlets Available

Six pamphlets in the U. S. Office of Education's National Defense Series are now off the press:

1. *What the Schools Can Do*, Pamphlet No. 4.
2. *Home Nursing Courses in High Schools*, Pamphlet No. 9.
3. *Hemisphere Solidarity*, Pamphlet No. 13.
4. *Education Under Dictatorships and in Democracies*, Pamphlet No. 15.
5. *How Libraries May Serve*, Pamphlet No. 17.
6. *Democracy in the Summer Camp*, Pamphlet No. 23.

What the Schools Can Do spotlights school services that can be strengthened to serve defense needs—services ranging from health and physical education and citizenship to international relations. Also emphasized are conservation of national resources, pupil guidance, and education for work. One section suggests how schools can organize to put into action specific defense-education recommendations.

How the schools can contribute directly to our national preparedness by instruction in home hygiene and care of the ill or injured is proposed in *Home Nursing Courses in High Schools*. The need for instruction, brief descriptions of selected courses, and vocational aspects of home nursing are presented.

Hemisphere Solidarity tells high-school teachers how they can help in the Good Neighbor program.

To help students, teachers, and school administrators compare education under dictator governments and in the United States, the Office has prepared *Education Under Dictatorships and in Democracies*. This pamphlet reviews the philosophies in education for fascism, communism, and naziism, and suggests what should be today's educational bulwarks of democracy.

Library service and methods of cooperation with the schools in promoting national preparedness are summarized in *How Libraries May Serve*. Addressed to school, college, public, and special libraries, this pamphlet lists the special problems libraries face in this emergency and how they can aid in meeting defense needs. Examples are reported and services recommended.

Democracy in the Summer Camp has been used quite widely during the past few months. It suggests ways of giving young people an opportunity to see democracy at work. While it is addressed particularly to those directing camp activities, the proposals are also useful to club and organization leaders.

More than 20 publications will be included in the U. S. Office of Education's National Defense Series. Other pamphlets in the series in press, or in preparation, will deal with such subjects as education for democracy, guidance problems, nutrition education, rural youth, Latin-American relationships, and contributions of voluntary agencies. An Office of Education committee, headed by Assistant U. S. Commissioner of Education, Bess Goodykoontz, is developing the series of pamphlet aids to help education meet defense problems.

A single copy price of 15 cents applies to the publications issued in the defense series to date. School officials and club leaders can take advantage of the special 25% discount on 100 copies or more sent to one address. Orders for the defense series publications should be sent to the Superintendent of Documents Washington, D. C.

Providence, R. I., Sept. 6.—Gratified with the achievements of its experimental summer session, Brown University will continue its defense education program in applied mechanics during the coming academic year, according to plans disclosed today by Dean Roland G. D. Richardson of Brown's Graduate School.

The program of instruction and research will be directed towards meeting what the National Research Council has described as a "critical need" for training experts who can apply higher mathematics to special problems in defense industries, especially aviation.

Like the summer session, the full year program will also be designed for advanced students who want to begin work in applied mathematics, either as research investigators or in preparation for teaching careers. No tuition will be charged for any accepted applicant.

Dean Richardson emphasized the importance of the work, the first full-time program of its kind in this country, by pointing out that since the war American industries can no longer share the results of research in applied mechanics which were carried out in Europe, particularly in Germany.

"At the same time," he said, "the war has greatly intensified the need for remedying Amer-

ica's inadequacies in industrial mathematics. A full exploitation of aerodynamics and other fields bearing directly upon defense activities awaits the basic work of mathematicians.

"There should be some center where men can obtain a broad training in the advanced reaches of mathematics applied to engineering, and where they can catch the spirit of research and learn the necessary techniques.

"Brown University wishes to serve the cause by offering such a program in a restricted field. This program has the double purpose of serving the nation's defense needs in the special realm of applied mechanics, and of pointing the way to a possible means of solving some of the more difficult engineering problems in industry."

All of the courses to be offered during the next two semesters, from Oct. 1 to Jan. 31 and from Feb. 11 to June 6, will be intensive and on a highly advanced plane. They include "Numerical and Graphical Methods in Applied Mathematics," "Partial Differential Equations," "Aerodynamics," "Elasticity," and seminars in various aspects of mechanics.

These courses will deal with specialized problems in mathematics which are related to aviation and airplane construction, with shipbuilding, gun manufacturing, armor-plating on tanks and cars, optical instruments, and other weapons and instruments of war requiring a high degree of technical knowledge to plan, design and make.

In addition to lectures, considerable time will be given over to individual and group conferences with members of the teaching staff. Some students are expected to bring with them problems requiring mathematical solutions. There will also be opportunities for laboratory testing of various theories.

The program is part of the Engineering Defense Training Program of the United States Office of Education, and has the added support of the Carnegie Corporation. Members of such organizations as the War Preparedness Committee of the American Mathematical Society, the Mathematical Association of America and the American Society of Mechanical Engineers have endorsed the project.

The teaching staff is now being organized, Dean Richardson said. He expects that some of the leading experts in charge of the summer courses can be retained, and that others will be added. There will be a total of at least four.

Among the present staff members are Richard von Mises, former director of the Institute for Applied Mathematics at the University of Berlin; Kurt O. Freidrichs, professor of mathematics at New York University; Ivan S. Sokolnikoff of the National Defense Research Committee; Jacob D. Tamarkin, professor of mathematics at Brown; and Stefan Bergmann of M.I.T.

The university has appointed to its staff Prof. Willi Prager of the University of Istanbul, an expert in the theory of structures. Prof. Prager has not yet arrived in this country.

The next annual meeting of the American Association of Junior Colleges will take place at Los Angeles, California, February 26–28, according to an announcement by Walter C. Eells, executive secretary of the association. Representatives of the nation's 650 junior colleges will travel to California for the event.

Robert G. Sproul, president of the University of California, is scheduled as main speaker at the three-day conclave. Chief emphasis of the meeting will be on defense in its relation to education and on terminal education activities. The association through its Commission on Junior College Terminal Education is now carrying on an extensive study of terminal education sponsored by the General Education Board of New York.

This will be the second time California has been chosen for the national junior college meeting. Eleven years ago in November 1930 when the junior college movement was just getting under way, the tenth annual meeting of the association was held in Berkeley. At that time there were some 54,000 young people enrolled in a few hundred junior colleges in the country; since then the enrollment has increased fourfold until today there are approximately 250,000 young men and women in 650 junior colleges.

Rosco C. Ingalls, president of Los Angeles City College, is chairman of the committee on arrangements. James C. Miller, president of Christian College, Columbia, Missouri, is president of the association. The 1941 meeting of the association was held in Chicago last February.

Local mathematics clubs may secure outstanding leaders in the field of mathematics en route to the National Council of Teachers of Mathematics meeting in San Francisco in February. Any club that is interested in securing such a speaker should communicate with THE MATHEMATICS TEACHER, 525 W. 120 St., New York, N. Y.

At Harvard University Mr. Lynn Harold Loomis of the Society of Fellows has been appointed Faculty Instructor in Mathematics.

The William Lowell Putnam Scholarship for 1941 for study at Harvard University has been

awarded to Richard F. Arens of the University of California at Los Angeles.

Fifth Annual Meeting of The Nebraska Section of the National Council of Teachers of Mathematics

University of Nebraska, Lincoln. Saturday, May 3, 1941, 9 A.M. Social Science Room 107 B.

Executive Board

Chairman, Milton Beckmann, Kearney

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Treasurer, Ruth Thompson, Minden
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N.C.T.M. Representative, Dr. A. R. Congdon, Lincoln

Program Theme: *Mathematics in Nebraska's Defense Program.*

PART I

1. Mathematical Requirements in the Program of the National Defense: W. C. Brenke, Head of the Mathematics Department, University of Nebraska. Lincoln, Nebraska.
2. Mathematics and Aviation: Carl R. Thomas, First President of the Nebr. Sec. of the N. C. T. of M. He taught navigation and meteorology under the C.A.A. last year at Chadron. At present he is teaching at the Lincoln Flying School. He is instructing army boys in shop mathematics.
3. Visual Aids in Mathematics: Inez M. Cook, teaches in the Lincoln Senior High School.
4. Curriculum Trends and the Implication for High School Mathematics: Miss Alma Wittenberger, North Platte, Nebraska, is chairman of Nebr. Mathematics Curriculum Committee of the N.C.T.M.

PART II

Joint Luncheon: Members of the National Council of Teachers of Mathematics and members of the Mathematical Association of America.

Luncheon Speakers: How Dr. Congdon viewed the mathematics meetings at Baton Rouge and Atlantic City. Professor H. M. Cox of the Institutional Research Bureau of the University, is to speak on the subject "Classification Tests in Mathematics."

Abstracts of Speeches

Professor W. C. Brenke

He will give us the plan under which the War Preparedness Committee of the two national mathematical societies is operating and suggest procedures under which high schools might lend substantial and effective assistance in the accomplishment of this plan. Some analysis will be made of the problems proposed by Professor Brenke in the May number of the Bulletin of the Nebraska Section of the National Council of Teachers of Mathematics.

Carl R. Thomas

"Increasing emphasis has been placed upon aviation as a most important development in connection with the defense program. Efficient training in mathematics is necessary as basic training to insure dependable construction, maintenance, and operation of planes.

"I am familiar with the mathematics involved in the navigation problems connected with the operation of planes. I am at present teaching Shop Mathematics to army men in training for plane maintenance.

"I shall present examples to show the nature of the mathematics involved in navigation problems and in shop problems. This information should aid high school mathematics teachers in creating interest in better preparation.',

Miss Alma Wittenberger

Miss Wittenberger received her Master's degree from Columbia University. She has written a paper entitled "An Approach to Advertising Through Geometry." This was written at the University of Columbia under the direction of Dr. Max Black, a visiting professor from the University of London. She has also had work under Dr. John R. Clark and Prof. W. C. Reeves. Miss Wittenberger will have much to offer in the way of curriculum trends.

Miss Inez M. Cook

Miss Cook has in mind visual aids in mathematics as seen at the Baton Rouge meeting. She will impart information concerning films shown at Baton Rouge, some visual geometry material, and fourth dimension.

Around the Luncheon Table

At the luncheon meeting at Atlantic City in February, 1941, a number of leaders at the twenty-four tables returned summaries of their discussions. The topics were exceedingly varied, and any attempt to mold all the reports into a coherent unified report had to be abandoned. Since these summaries were the result of the combined thinking of all participants, it was

felt that the report should avoid personalities, and let the conclusions, problems, questions and suggestions speak for themselves.

The pupils who cannot do mathematics or do not do mathematics have been pulling the level of passing work downward, with the result that capable students are loafing and getting by. For the truly slow pupils, teaching can be made effective if it touches their lives. For this, it is necessary:—to know the home life, secured by personal visitation on the part of the teacher; to have small sized classes; to learn the interest of the children; to develop competition in learning; to let the child feel, see, hold, and play with mechanical models of mathematical principles.

To provide for the present heterogeneous high school population, a two-lane highway in the mathematics curriculum is necessary. Some possible lanes are socio-economic; college preparatory: vocational: traditional; enriched courses for bright students. Provision must be made, no matter what lanes are adopted, for changing a pupil from one lane to the other. Proper guidance into these lanes from the outset is essential.

The defense program as outlined by Dr. Hart's report raises the following questions concerning the mathematics curriculum:

How much solid geometry and trigonometry should be taught in the high school?

Should all students be given a minimum amount of mathematical training?

Should astronomy and spherical trigonometry be reintroduced into the high school?

What mathematical training is needed by all boys who will enter the defense program?

In Connecticut's defense program, of the 200 hours spent in training workers, 20 hours are given to related mathematics. Girls entering the aeroplane factories as inspectors must have had four years of high school mathematics. The mechanical and technical aspects of a defense program make a knowledge of mathematics, including trigonometry and solid geometry, indispensable.

The important problems concerning department meetings are: what, and how many, topics should be discussed, and who shall select them? It was the opinion that one topic, with sub-topics built around it shall form a nucleus for the year's study, and that all topics should be selected by preference of the teachers, not the head. These meetings should be given over in greater part to a study of teaching problems and not petty administrative problems.

The mathematics curriculum should be the same for all students from grades 7 to 10 and be built on a problem basis. These problems should include all mathematics as one general course and treated from three levels: (1) thinking in terms of simple relationships; (2) discovering relationships; (3) thinking in terms of abstractions. The eleventh and twelfth year should be devoted to specialization in accordance with these three levels.

The following questions remained unanswered as to the vital thing in high school mathematics:—Should mathematics be taught as 'Pure Mathematics'? How much transfer should there be in teaching 'Pure Geometry'? Is measurement as valid a method of arriving at satisfactory conclusions as proceeding from a particular set of assumptions with a logical proof?

The geometry of the school is too idealistic, too perfect, too simple as compared with thinking in life situations. Hence, we should stress functional or relational thinking to gain as much transfer as possible.

Permanent learning, as viewed from Gestalt Psychology, can be established through configurations. There are several ways to build configurations. Among the most important are those built around a social situation, and around a mathematical concept.

The importance of a configuration in the teaching of mathematics is dependent on the contribution it makes to the child's mathematical education, either in a clearer understanding of concepts or in a clearer knowledge of the uses of quantitative thinking. A configuration loses its value to the extent that its contribution to the child's education becomes secondary to the development of a procedure.

A course in statistics in the high school should be informational and interpretive and not for the purpose of making thorough computers of statistical constants. Such a course may probably be best combined with consumer mathematics in the twelfth year.

The important concepts to be included in such a course are distributions; measurements; averages; simple deviations; simple correlations and relationships. The important problem for research is to select for use such data as are real, practical, and of value to the pupil.

Teachers should encourage pupils to investigate through experience, but to distrust

hunches. Logical proofs should be sought for all hunches. Thus, the fourth "R" in education should be "Reasoning." Each large school system should employ a logician to act as coordinator of topics in reasoning selected from all academic subjects. Examples of correct logic, as well as, errors in reasoning, should be used in geometry classes to equip pupils for correct reasoning in life.—Howard F. Fehr.

On April 11, the Mathematics Section of the Tennessee Education Association held its annual meeting at the Hume-Fogg High School, Nashville, Tennessee. Ninety-seven teachers of mathematics attended. The following program was given:

Presiding—Mr. Milton Nixon, President, Monterey; Mrs. Sarah Branddon, Vice-President, Dover; Miss Leekie Smith, Secretary, Nashville.

"The Mathematics Needed in the Science Courses," Dr. A. W. Dicus, Tennessee Polytechnic Institute, Cookeville, Tennessee.

"Weaknesses in Mathematics in College Freshmen," Mr. Albert R. Sloan, Carson-Newman College, Jefferson City, Tennessee.

"The Need of Mathematics in the Industrial Fields," Mr. E. D. MacArthur, Supervisor of Mechanical Assistance Department, E. I. Du Pont de Nemours and Company, Old Hickory, Tennessee.

"The Role That Mathematics Can Play in the Preparedness Program," Captain M. H. Stopinski, Castle Heights Military Academy, Lebanon, Tennessee.

A number of mathematic displays were exhibited from mathematics classes of the State of Tennessee.—Milton Nixon.

An Institute for Teachers of Secondary Mathematics was held at Duke University, Durham, N. C. in co-operation with The Mathematical Section of the North Carolina Education Association from June 17 to June 20, 1941.

Tuesday, June 17

3:30 P.M.: *Presiding: Professor W. W. Rankin*
"Some Practical Problems of Secondary Mathematics Whose Solution Requires a Knowledge of Algebra and Geometry"
In Business: Professor E. T. Browne, University of North Carolina
In Industry: Dr. F. G. Dressel, Duke University
In Science: Professor Helen Barton, W. C.-U. N. C., Greensboro
Discussion

6:30 P.M.: Dinner (informal dress), Social Room, Union

Presiding: Professor J. J. Gergen, Duke University
Welcome: Dr. R. L. Flowers, President of Duke University
"A Toast to Teachers of Mathematics"
Professor Archibald Henderson, University of North Carolina
"Some Fundamental Concepts in Secondary Mathematics"
Professor F. D. Murnaghan, Johns Hopkins University
Discussion

9:00 A.M.–10:00 P.M.: Social Hour

Wednesday, June 18

10:30 A.M.: *Presiding: Miss Olive Smith, Reynolds High School, Winston-Salem*
Committee: Miss Eura Strother, Junior High School, Durham, Chairman
Miss Laura Efird, Hugh Morson High School, Raleigh
Miss Bonnie Cone, Central High School, Charlotte
Miss Margaret Ricks, Rocky Mount High School
Miss Lillie Walker, Greensboro High School

11:00 A.M.: "Mathematical Models and Their Use in the Study of Mathematics"
Professor Ruth Stokes, Winthrop College
Discussion

12:30 P.M.: Adjournment

3:30 P.M.: *Presiding: Professor K. B. Patterson, Duke University*
"The Demonstration of the Slide Rule"
Mr. Glen Hayden, Jr. (student), Chapel Hill High School
Mr. John Shelden, Hugh Morson High School
"Field Work and Use of Instruments in the Study of Mathematics"
Professor W. J. Seeley, Duke University
High School Teachers and Students
Discussion

5:30 P.M.: Adjournement

8:00 P.M.: *Presiding: Professor H. A. Fischer, State College*
"Mathematics of Artillery Fire"
Colonel F. M. Crist, Fort Bragg
"Mathematics for Defense"
Professor J. M. Thomas, Duke University
Discussion

10:00 P.M.: Adjournment

Thursday, June 19

3:30 P.M.: *Presiding: Professor E. T. Browne, University of North Carolina*
"Traditional versus Unified Mathematics"
Panel Discussion:
Traditional: Mr. A. R. Joyner, Reynolds High School, Winston-Salem
Dr. J. M. Clarkson, North Carolina State College
Unified: Miss Annie Anderson, Chapel Hill High School
Dr. V. A. Hoyle, University of North Carolina

5:00 P.M.: Tea, Graham Memorial

8:00 P.M.: *Presiding: Professor E. F. Canaday, Meredith College*
"Recent Literature in Mathematics Helpful to Teachers and Students"
Miss Laura Efird, Hugh Morson High School, Raleigh
"The Place of Mathematics in Secondary Education"
(A report of the Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics)
Professor H. F. Munch, University of North Carolina
Professor J. W. Lasley, University of North Carolina
Discussion

10:00 P.M.: Adjournment

Friday, June 20

3:30 P.M.: *Presiding: Professor J. C. Carroll, Wake Forest College*
"Mathematics Preparatory for College Mathematics"
Professor E. Cameron, University of North Carolina
Professor W. G. McGavock, Davidson College
"Mathematics for Students Not Going to College"
Mr. John L. Woodward, Durham High School
Miss Bonnie Cone, Central High School, Charlotte
Discussion

5:30 P.M.: Adjournment

6:30 P.M.: Dinner, *Presiding: Professor W. W. Rankin*
"The Satisfaction and Fun of Teaching"
Professor B. G. Childs, Duke University
Discussion
Mathematical Movie (to be selected)

9:20–10:00 P.M.: Dance, Social Room, Union

Exhibit

High schools were invited to make exhibits at the Institute. The exhibits included such items as:: models, instruments (preferably home made), maps, drawings, scrap-books (only one from each school), lists of mathematical books in library of school, textbooks more than 50 years old, etc.

Prize

The Alumni Office of Duke University gave a prize of $25.00 (in mathematical equipment) to the high school which made the best exhibit at the Institute.

Entertainment Committee

Dr. F. G. Dressel, *Chairman*
Duke University

Miss Louise Watkins
Durham High School

Mr. B. L. DeBruyne
Durham High School

Mrs. Rosalie Elliott
Durham High School

Miss Eleanor Rankin
Durham High School (student)

Mr. Donald Rose
Durham High School (student)

Mr. Glen Hayden, Jr.
Chapel Hill High School (student)

Professor J. A. Greenwood
Duke University

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Line Motion Linkages (How to Draw a Straight Line)
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